Construction of the moduli space of stable parabolic Higgs bundles on a Riemann surface

By Hiroshi KONNO

0. Introduction.

In [16] Narasimhan and Seshadri proved that every stable vector bundle on a compact Riemann surface comes from an irreducible projective unitary representation of the fundamental group. In other words there exists an irreducible Hermitian Einstein metric on every stable vector bundle. In [2] Atiyah and Bott observed that the moduli space of stable vector bundles is considered as a Kähler quotient of the space of all holomorphic structures by the gauge group. In [6] Donaldson gave a different proof of the theorem of Narasimhan and Seshadri in this context. This theorem was generalized to higher dimensional cases by Donaldson [7, 8] and Uhlenbeck and Yau [19].

In [11] Hitchin extended this theory in another direction. He introduced the notion of a Higgs bundle, which is a generalization of a holomorphic vector bundle. He also introduced the notion of stability for Higgs bundles and showed that there exist irreducible Hermitian Einstein metrics on stable Higgs bundles and that stable Higgs bundles correspond to irreducible projective (not necessary unitary) representations of the fundamental group. He also pointed out that the moduli space of stable Higgs bundles can be viewed as a hyperkähler quotient. In [17] Simpson generalized this result to higher dimensional cases.

It seems quite natural to generalize these results to the case when base spaces are noncompact. In [15] Mehta and Seshadri introduced the notion of a parabolic vector bundle, which is a pair of a vector bundle and flags of fibers over some points called parabolic points. They showed that every stable parabolic vector bundle comes from an irreducible projective unitary representations of the fundamental group of the complement of the parabolic points. In [3] Biquard gave a different proof of this theorem from a gauge theoretical point of view by choosing the appropriate Sobolev completion of the space of all parabolic holomorphic structures.

In the case of stable parabolic Higgs bundles Simpson [17] showed the existence of Hermitian Einstein metrics. To do this, he solved a PDE in some