J. Math. Soc. Japan
Vol. 45, No. 2, 1993

On the integrated density of states for the Schrödinger operators with certain random electromagnetic potentials

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(Received Dec. 17, 1991)

1. Introduction.

Let $\{V(x, \omega);\ x\in \boldsymbol{R}^d\}$ and $\{A(x, \omega);\ x\in \boldsymbol{R}^d\}$ be mutually independent, stationary random fields with values in $\boldsymbol{R}$ and in $\boldsymbol{R}^d$, respectively, defined on a probability space $(\Omega, \mathcal{F}, P)$. In this article we are interested in a family $\{H(\omega)\}_{\omega\in\Omega}$ of the Schrödinger operators with magnetic fields which are defined by

$$H(\omega) = \frac{1}{2}\sum_{j=1}^{d}\left(\sqrt{-1}\frac{\partial}{\partial x^j} - A_j^\omega\right)^2 + V^\omega, \tag{1.1}$$

where A_j^ω and V^ω are the multiplication operators given by $A_j^\omega(x)=A_j(x, \omega)$ and $V^\omega(x)=V(x, \omega)$, respectively. $A^\omega=(A_1^\omega, A_2^\omega, \cdots, A_d^\omega)$ is called the (magnetic) vector potential and is often identified with the differential 1-form $\theta^\omega=\sum_{j=1}^d A_j^\omega(x)dx^j$. Then the corresponding magnetic field is given by the differential 2-form $d\theta^\omega$ and, if $d=1$, $H(\omega)$ is unitary equivalent to the operator $-(1/2)d^2/dx^2+V^\omega$, for which fairly satisfactory theory has been developed. Therefore we throughout assume that $d\geqq 2$. As usual we call V^ω the (electric) scalar potential.

Such Schrödinger operators arise in quantum models of disordered systems (see, e.g., [5], [9], [11], [13], [17] and references therein) and the spectral properties are one of the main targets of the study. Among them we are interested in the integrated density of states (IDS for short).

The IDS is defined as follows. Let Λ_r, $r=0, 1, 2, \cdots$ be a sequence of hypercubes increasing to $\boldsymbol{R}^d$ and $H_{\Lambda_r}^D(\omega)$ be the restriction of $H(\omega)$ given by (1.1) to $L^2(\Lambda_r)$ with the Dirichlet boundary condition. We denote by $\rho_{\Lambda_r}^D(\lambda, \omega)$ the number of the eigenvalues less than λ of $H_{\Lambda_r}^D(\omega)$. Then the IDS ρ^D is defined by

$$\rho^D(\lambda) = \lim_{r\to\infty}\frac{1}{|\Lambda_r|}\rho_{\Lambda_r}^D(\lambda, \omega), \tag{1.2}$$

where $|A|$ is the volume of a subset A in $\boldsymbol{R}^d$. Under some natural assump-