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Mass minimizing submanifolds with respect to some Riemannian metrics

Dedicated to Professor Hisao Nakagawa on his sixtieth birthday

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Introduction.

We characterize mass minimizing submanifolds with respect to some Riemannian metrics in this paper. If a compact oriented submanifold M is mass minimizing in its real homology class with respect to some Riemannian metric on X, then M is not homologous to 0 in $H_*(X;\boldsymbol{R})$, where we consider the homology classes consisting of currents and regard a compact oriented submanifold as a current. The mass of a current is defined in Section 1, which is a generalization of the usual volume of a Riemannian manifold. We shall prove a converse statement.

THEOREM 1. *Let M be a compact oriented submanifold embedded in a manifold X. If the real homology class represented by M in X is not equal to* 0, *then there exists a Riemannian metric g on X such that M is mass minimizing in its real homology class with respect to g.*

This will be proved in Section 2. We shall show the following corollary of Theorem 1.

COROLLARY 2. *Let M be a compact oriented submanifold embedded in a manifold X. Then there exist a Riemannian metric g on X and a neighborhood N of M such that* $\mathrm{vol}_g(M)\leqq\mathrm{vol}_g(M_t)$ *for any variation M_t of M in N with $M_0=M$.*

Sullivan [**Su2**] and Harvey-Lawson [**H-L1**] characterized foliations consisting of minimal leaves and homologically mass minimizing leaves with respect to some Riemannian metrics respectively. We shall use their methods to prove Theorem 1.

We also consider an equivariant version of Theorem 1. In fact we shall prove the following theorem and corollary in Section 3.

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