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On the subalgebras $\mathfrak{g}_0$ and $\mathfrak{g}_{ev}$ of semisimple graded Lie algebras

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Introduction.

Graded Lie algebras (abbreviated as GLA's in the sequel), even those of finite dimensions, play important roles in many fields in mathematics. In this article we shall always assume that Lie algebras are defined over $\boldsymbol{R}$ and are finite-dimensional. Let $\mathfrak{g}=\sum_{k\in Z}\mathfrak{g}_k$ be a semisimple GLA. Then there exists an integer $\nu\geqq 1$ such that $\mathfrak{g}_{\pm\nu}\neq(0)$ and $\mathfrak{g}_k=(0)$ for $|k|>\nu$. In this case we say that the GLA $\mathfrak{g}$ is *of the ν-th kind*. The family of the subspaces $(\mathfrak{g}_k)_{-\nu\leqq k\leqq\nu}$ is called a *gradation* on the Lie algebra $\mathfrak{g}$. Classifying GLA's is obviously equivalent to classifying gradations on each Lie algebra. We use the word "classification" in two ways. By a *weak classification* (or simply a *classification*), we mean the construction of a bijection between the set of isomorphism classes of gradations on a given Lie algebra $\mathfrak{g}$ and a certain set which is more easily accessible. By the *strong classification* we mean a weak classification plus the explicit determination of the subspaces $\mathfrak{g}_k$ of gradations. In our previous paper [**11**], we gave a classification of gradations on a semisimple Lie algebra $\mathfrak{g}$ in terms of its restricted fundamental root system. The same problem was treated also in Djokovič [**5**] from a slightly different point of view (See also Z. Hou [**7**]). Among semisimple GLA's, those which are most important for applications are GLA's of the first kind and of the second kind. In this direction, Kobayashi-Nagano [**12**] gave the strong classification of classical simple GLA's of the first kind and a classification of exceptional simple GLA's of the first kind. The strong classification of exceptional simple GLA's of the first kind was made by O. Loos [**13**]. J.H. Cheng [**4**] gave a classification of simple GLA's of the second kind under the condition that $\dim\mathfrak{g}_{-2}=1$. Afterwards, we gave in [**11**] a classification of classical simple GLA's of the second kind (without any assumptions) and determined the subspaces $\mathfrak{g}_{-1}$ for each case.

In the present paper, we give the strong classification of classical simple GLA's of the second kind and a classification of exceptional simple GLA's of

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