J. Math. Soc. Japan
Vol. 44, No. 4, 1992

A remark on the Ochanine genus

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(Received Sept. 25, 1991)

Introduction.

In a celebrated paper [13], Witten asserted the rigidity theorem for the elliptic genus, which is an extension of the Atiyah-Hirzebruch theorem concerning the $\hat{A}$-genus. The elliptic genus is a formal power series in q whose coefficients are the indices of the Dirac operators coupled with certain vector bundles associated to the tangent bundle. After some partial results of Landweber, Stong and Ochanine, Bott and Taubes gave a mathematically rigorous proof to the rigidity theorem [3].

Recently Ochanine defined the KO-version of the elliptic genus [8]. Bendersky and Ochanine proved that the Ochanine genus vanishes for spin manifolds admitting S^1-actions of odd type [2], [9]. In a previous paper [11], we proved that the α-invariant (which is called the Atiyah invariant in [2]) vanishes for spin manifolds admitting S^1-actions of odd type. The proof in [2], [9] is based on purely topological argument and we dealt with the analytical index of the real Dirac operator in [11]. The purpose of this note is to prove the vanishing result for the Ochanine genus by the method in [11]. Our method can also be applied to spin manifolds admitting involutions of odd type.

The author is grateful to Professor Mikiya Masuda for pointing out that the method in [11] does work for $8k+1$-dimensional spin manifolds admitting involutions of odd type. He also thanks Professor Hirzebruch for his comment of the Proposition in Appendix.

1. The Ochanine genus.

In this section we review the definition of the Ochanine genus [8] and the rigidity theorem of Witten [3], [13].

For a real vector bundle $E \to X$, we set

$$\Lambda_t(E) := \sum_{i \geq 0} \Lambda^i(E)t^i\,, \qquad S_t(E) := \sum_{i \geq 0} S^i(E)t^i$$