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Applications of spreading models to regular methods of summabilities and growth rate of Cesàro means

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§ 0. Introduction.

Applications of Brunel-Sucheston's spreading model are presented. One application is to show an alternative theorem concerning weakly null sequences of Banach spaces, another application is to show an alternative theorem of summabilities of bounded sequences in Banach spaces and the other one is to estimate, from above or below, the growth rate of Cesàro means.

1. One application of Brunel-Sucheston's spreading model is to show that each weakly null sequence of Banach spaces has a subsequence which is either "uniformly completely Cesàro summable" or "uniformly completely non Cesàro summable".

THEOREM I. *For every weakly null sequence* $\{x_n\}_n$ *of a Banach space* X, *one can extract a subsequence* $\{x'_n\}_n$ *of* $\{x_n\}_n$ *such that either*

$$\lim_{k\to\infty}\Big(\sup_{n_1<\cdots<n_k,\,|a_i|\leq 1}\Big\|\frac{1}{k}\sum_{i=1}^{k} a_i x'_{n_i}\Big\|\Big)=0 \tag{1}$$

or

$$\inf_k\Big(\inf_{n_1<\cdots<n_k,\,|\theta_i|=1}\Big\|\frac{1}{k}\sum_{i=1}^{k}\theta_i x'_{n_i}\Big\|\Big)>0\,. \tag{2}$$

2. A real or complex infinite matrix $(a_{n,m})_{n,m}$ defines *a regular method of summability*, if (and only if) the following conditions hold:

$$\sup_n\Big(\sum_{m=1}^{\infty}|a_{n,m}|\Big)<\infty\,, \tag{1}$$

$$\lim_{n\to\infty}\Big(\sum_{m=1}^{\infty}a_{n,m}\Big)=1 \tag{2}$$

and

$$\lim_{n\to\infty}a_{n,m}=0 \qquad (m\in\boldsymbol{N})\,. \tag{3}$$

Let Λ denote the set of all regular methods of summability and put

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