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The invariant differential forms on the Teichmüller space under the Fenchel-Nielsen flows

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Introduction.

We begin by giving an explanation of our motivation. Given a C^∞ manifold M, $\Omega^*(M)$ denotes the differential graded algebra of all C^∞ differential forms on M. Let G be a semi-simple Lie group, K its maximal compact subgroup, and Γ a discrete subgroup of G. The inclusion of invariant algebras

$$\Omega^*(G/K)^G \subset \Omega^*(G/K)^\Gamma = \Omega^*(\Gamma\backslash G/K)$$

induces the homomorphism

$$H^*(\Omega^*(G/K)^G) \longrightarrow H^*(\Gamma\backslash G/K).$$

In the case Γ is cocompact, Matsushima [**Ma**] established that it is isomorphic for sufficiently small $*$. Using this homomorphism Borel [**B**] determined the stable real cohomology of arithmetic groups.

Our purpose is to find an analogue of this Matsushima's theory for the moduli space of compact Riemann surfaces of genus g, $\mathbf{M}_g$. More precisely we want to find out a group which transitively acts on the Teichmüller space of compact Riemann surfaces of genus g, T_g, and includes the mapping class group $\mathcal{M}_g$ as a "discrete" subgroup. Then we want to investigate the invariant differential forms on T_g under the group and its relation to the real cohomology of $\mathbf{M}_g$.

On the other hand, by Dehn-Lickorish [**L**], the mapping class group $\mathcal{M}_g$ is generated by the Dehn twists associated to the simple closed curves in the oriented closed surface of genus g, Σ_g. The twist associated to a simple closed curve α is described as follows; cut Σ_g along α, rotate one boundary relative to the other in the angle 2π, and reglue in this new position. Varying the rotation angle over the real numbers, Fenchel-Nielsen defined the flow on the Teichmüller space T_g, called the Fenchel-Nielsen flow associated to α [**W1**][**G2**]. When we define FN the subgroup of $Diff(T_g)$ generated by the Fenchel-Nielsen flows associated to all simple closed geodesics, FN includes $\mathcal{M}_g$ as a "discrete" subgroup. Furthermore the group FN acts on T_g transitively in view of a