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Einstein-Hermitian connections on Hyper-Kähler quotients

Dedicated to Professor Tadashi Nagano on his 60th birthday

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1. Main result.

A *hyper-Kähler structure* on a Riemannian manifold (Y, g) is a set of three almost complex structures (I, J, K) which are parallel with respect to the Levi-Civita connection and satisfy the quaternion relations

$$IJ = -JI = K.$$

We have the associated Kähler forms $\omega_I, \omega_J, \omega_K$ defined by

$$\omega_I(v, w) = g(Iv, w), \qquad \omega_J(v, w) = g(Jv, w),$$
$$\omega_K(v, w) = g(Kv, w), \qquad \text{for } v, w \in TY$$

which are closed and parallel.

Let G be a compact Lie group acting on Y so as to preserve the metric g and the hyper-Kähler structure (I, J, K). Each element $\xi \in \mathfrak{g}$ of the Lie algebra of G defines a vector field ξ^* on Y which generates the action of ξ. The hyper-Kähler moment map defined below is the set of three moment maps.

DEFINITION 1.1. *A hyper-Kähler moment map* for the action of G on Y is a map $\mu = (\mu_I, \mu_J, \mu_K): Y \to \boldsymbol{R}^3 \otimes \mathfrak{g}^*$ which satisfies

(a) $$\mu_A(y \cdot g) = \mathrm{Ad}_g^*(\mu_A(y)), \qquad y \in Y,\ g \in G,\ A = I, J, K$$

(b) $$\langle \xi, d\mu_A(v) \rangle = \omega_A(\xi^*, v), \qquad v \in TY,\ \xi \in \mathfrak{g},\ A = I, J, K,$$

where $\mathfrak{g}^*$ is the dual space of $\mathfrak{g}$, $\mathrm{Ad}^*: \mathfrak{g}^* \to \mathfrak{g}^*$ is the coadjoint map and $\langle\ ,\ \rangle$ denotes the dual pairing between $\mathfrak{g}$ and $\mathfrak{g}^*$.

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