J. Math. Soc. Japan
Vol. 44, No. 1, 1992

Growth properties of p-th hyperplane means of Green potentials in a half space

By Yoshihiro MIZUTA

(Received Sept. 12, 1990)

1. Introduction.

Recently, for a Green potential v on the unit ball of $\boldsymbol{R}^n$, Gardiner [1] studied the limiting behavior of $\mathcal{M}_p(v, r)$, which is the p-th order mean of v over the sphere of radius r centered at the origin. In this paper we are concerned with Green potentials Gf in the half space $D=\{x=(x', x_n)\in\boldsymbol{R}^{n-1}\times\boldsymbol{R}^1; x_n>0\}$, where $n\geqq 2$ and f is a nonnegative measurable function on D satisfying

$$\int_D x_n^\alpha f(x)^q dx < \infty,$$

where $q\geqq 1$ and $\alpha\leqq 2q-1$. For $p>0$ and a nonnegative Borel measurable function u on D, define $M_p(u, r)=\left(\int_{R^{n-1}} u(x', r)^p dx'\right)^{1/p}$; in case $p=\infty$, define $M_\infty(u, r)=\sup\{u(x', r); x'\in\boldsymbol{R}^{n-1}\}$. Our aim in this paper is to prove that

$$\lim_{x_n\downarrow 0} x_n^{(n-2q+\alpha)/q-(n-1)/p} M_p(Gf, x_n)=0,$$

or, more weakly,

$$\liminf_{x_n\downarrow 0} x_n^{(n-2q+\alpha)/q-(n-1)/p} M_p(Gf, x_n)=0$$

for p satisfying a suitable condition; the power of x_n is shown to be best possible. In case $q=1$, our theorems below give versions of Gardiner's results in [1] to the half space.

2. Preliminary lemmas.

Now we give some notation and terminologies needed later. Let $G(x, y)$ denote the Green function in the half space D, that is,

$$G(x, y)=\begin{cases} |x-y|^{2-n}-|\bar{x}-y|^{2-n} & \text{in case } n\geqq 3, \\ \log(|\bar{x}-y|/|x-y|) & \text{in case } n=2, \end{cases}$$

where $\bar{x}=(x', -x_n)$ for $x=(x', x_n)$. We define the Green potential $G\mu$ of a nonnegative (Radon) measure μ on D by setting