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Structure and duality of $\mathscr{D}$-modules related to KP hierarchy

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1. Introduction.

Nonlinear integrable systems, as pointed out by Sato [**10**, **11**], may be understood as "deformations" of $\mathscr{D}$-modules. For the case of the KP hierarchy, such a $\mathscr{D}$-module is given by

$$\mathscr{M}_0 = \mathscr{D}_0 W ,$$

where W is a pseudo-differential operator of the form

$$W = 1 + \sum_{m=1}^{\infty} w_m(t, x)\partial^{-m} , \qquad \partial = \partial/\partial x ,$$

$\mathscr{D}_0$ is a ring of ordinary differential operators in x, and $w_m(t, x)$ are unknown functions of the KP hierarchy [8, **9**] that depend on an infinite number of "time variables" $t=(t_1, t_2, \cdots)$. In algebraic approaches [8, **9**, **6**], it has been customary to consider "formal" (or "formally regular") solutions, i.e., solutions for which

$$w_m(t, x) \in \boldsymbol{C}[[t, x]] .$$

In that setting, $\mathscr{D}_0$ is given by

$$\mathscr{D}_0 = \{P;\ \exists m,\ P = \sum_{n=0}^{m} p_n\partial^n,\ p_n \in \boldsymbol{C}[[t, x]]\}.$$

This is a subring of the ring

$$\mathscr{E}_0 = \{P;\ \exists m,\ P = \sum_{n=-\infty}^{m} p_n\partial^n,\ p_n \in \boldsymbol{C}[[t, x]]\}$$

of pseudo-differential operators. $\mathscr{M}_0$ is thus a free left $\mathscr{D}_0$-submodule of $\mathscr{E}_0$. With these notions, Sato [**10**, **11**] presented an abstract formulation of the KP hierarchy and its relation to the "universal Grassmannian manifold" (UGM) [8, **9**].

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