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A decomposition theorem in a Banach $*$-algebra related to completely bounded maps on C^*-algebras

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1. Introduction.

As one of the fundamental theorems in C^*-algebras, it is well known that a self-adjoint element has the Jordan decomposition and a self-adjoint bounded linear functional has the Hahn decomposition: i.e. if x is a self-adjoint element in a C^*-algebra A, then there exist two positive elements $x_1, x_2 \in A$ such that $x=x_1-x_2$, $x_1x_2=0$ and $\|x\|=\|x_1+x_2\|$. If f is a self-adjoint bounded linear functional on A, then there exist two positive linear functionals $f_1, f_2 \in A^*$ such that $f=f_1-f_2$ and $\|f\|=\|f_1+f_2\|$.

As a generalized version of the Hahn decomposition, Loebl and Tsui considered independently whether the bounded self-adjoint map has the positive decomposition [**10**], [**16**]. The answer was negative except a few cases. Furthermore Huruya and Tomiyama obtained a non-existence theorem of the Hahn decomposition of bounded maps in the general situation [**8**]. However, it was Wittstock who showed the self-adjoint completely bounded map of a C^*-algebra to an injective C^*-algebra can be written as a difference of two completely positive maps with the norm condition [**17**]. This can be seen as a generalized Hahn decomposition, since the complete boundedness coincides with the boundedness and the complete positivity coincides with the positivity if the range algebra is commutative.

On the other hand, a completely bounded map can be regarded as an element in the dual space of a certain Banach space [**6**], [**9**], [**5**]. In this paper, especially motivated by the isomorphism which is obtained by Effros and Exel [**5**], we intend to get the Jordan decomposition and the Hahn decomposition in an advanced form. The main theorem is the following.

THEOREM B. *Let A be a C^*-algebra and $B(H)$ be all bounded operators on a Hilbert space H. Suppose that p is a finite dimensional projection.*

(1) *If V is a self-adjoint element in $pB(H)\otimes_h A\otimes_h B(H)p$ with the Haagerup norm $\|\ \|_h$, then*