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On cutting pseudo-foliations along incompressible surfaces

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A compact orientable irreducible 3-manifold containing a two-sided incompressible surface is called a *Haken manifold* ([2]). The Haken manifold has a hierarchy. I.e. we obtain cubes with handles by cutting along incompressible surfaces several times. We would like to investigate the relationship between the foliations without compact leaves and the hierarchies of the Haken manifolds.

In this paper, we introduce the notion of pseudo-foliations of a compact 3-manifold M and would like to give a condition for a pseudo-foliation $\mathcal{F}$ of M, on which there exists an incompressible surface S not parallel to ∂M such that the "foliation" obtained by cutting $\mathcal{F}$ along S is again a pseudo-foliation.

The notion of pseudo-foliations is used in [3] for classifying some transversely affine foliations $\mathcal{F}$ of some surface bundles M over S^1 with fiber Σ, where the main theorem of this paper is used for cutting $\mathcal{F}$ along incompressible surfaces into pseudo-foliations of cubes with handles. More precisely, $\mathcal{F}$ is first cut along some fiber into a pseudo-foliation of $\Sigma\times[0, 1]$ so that $\mathcal{F}|(\Sigma\times\{0\})$ is equal to $\mathcal{F}|(\Sigma\times\{1\})$ and $\mathcal{F}|(\Sigma\times[0, 1])$ has a transverse invariant measure by the conditions of M and $\mathcal{F}$. Next we cut $\mathcal{F}|(\Sigma\times[0, 1])$ along disjoint annuli into pseudo-foliations of cubes with handles. Since these pseudo-foliations with invariant measures can be classified, we obtain the classification of $\mathcal{F}$.

1. Definitions and results.

Let M be a compact orientable 3-manifold. A *pseudo-foliation* $\mathcal{F}$ of M is a transversely orientable C^2 foliation transverse to ∂M except at finitely many points $p_i\in\partial M$ $(i=1, 2, 3, \cdots, n)$ where p_i is a saddle singularity of $\mathcal{F}|\partial M$ and $\mathcal{F}|\partial M$ has no leaves connecting distinct saddle singularities. A simple closed curve on ∂M consisting of saddle singularities and leaves of $\mathcal{F}|\partial M$ is called a *cycle* of $\mathcal{F}|\partial M$ (A closed leaf is a cycle).

Let Σ be a compact connected orientable 2-manifold with boundary, and let ϕ, an embedding of Σ in M, where $\phi(\Sigma)\cap\partial M=\phi(\partial\Sigma)$. We put