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Newton polygons and formal Gevrey indices in the Cauchy-Goursat-Fuchs type equations

Dedicated to Professor Mutsuhide Matsumura on his 60th birthday

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Introduction.

In a recent paper [14], Yonemura has studied divergent formal power series solutions of the Cauchy problem to non Kowalevskian equation, and characterized its formal Gevrey index by a Newton polygon associated with the operator. It is a version to partial differential equations of results in ordinary differential equations studied by Ramis [12], where an index theory of ordinary differential operators in a category of formal Gevrey functions was studied.

In this paper, we shall extend and give refinements of Yonemura's result to integrodifferential operators, which we call of Cauchy-Goursat-Fuchs type. Precisely, we shall study a unique solvability of integrodifferential equations in a category of convergent power series or formal power series with formal Gevrey index. It should be mentioned that our main interest is in the divergent formal solutions, but the results obtained in the category of holomorphic functions are new.

It is well known that the Cauchy-Kowalevski theorem does not hold for non Kowalevskian equations. Precisely, the formal power series solution of the Cauchy problem to non Kowalevskian equation does not converge in general even if the Cauchy data and the right hand side of the equation converge. This was the main subject of the inverse problem of Cauchy-Kowalevski theorem (c.f. Miyake [8] and Mizohata [10]).

Nevertheless, we shall study the divergent formal power series solutions. To make clear the motivation of the study, we shall give two examples below.

EXAMPLE 0.1. Let us consider the Cauchy problem,

$$D_t u = t^{\sigma} D_x{}^m u, \qquad u(x, 0) = \varphi(x) \equiv \frac{1}{1-x}, \tag{0.1}$$

where $(x, t) \in \boldsymbol{C}^2$, $(D_x, D_t) = (\partial/\partial x, \partial/\partial t)$ and σ $(\geqq 0)$ and m $(\geqq 2)$ are integers. Then this Cauchy problem does not have a holomorphic solution in any neighbourhood of the origin. Indeed, (0.1) has a unique formal solution,