J. Math. Soc. Japan
Vol. 43, No. 1, 1991

Isometrical identities for the Bergman and the Szegö spaces on a sector

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(Received Oct. 9, 1989)
(Revised March 5, 1990)

1. Introduction.

Let $\Delta(\alpha)=\{z; |\arg z|<\alpha\}$. We consider the Bergman space

$$B_{\Delta(\alpha)}=\{F; F \text{ is analytic on } \Delta(\alpha),\ \|F\|_{B_{\Delta(\alpha)}}<\infty\},$$

where

$$\|F\|_{B_{\Delta(\alpha)}}=\left\{\iint_{\Delta(\alpha)}|F(x+iy)|^2 dxdy\right\}^{1/2}.$$

In the case of $\alpha=\pi/4$ we showed that $\|F\|_{B_{\Delta(\alpha)}}^2$ is represented as a series of weighted square integrals of the derivatives of the trace of F on the positive real axis ([2]). The proof included two different ingredients: an integral transform and a heat equation on the positive real axis. Both of them required rather deep and lengthy arguments which worked only in the case of $\alpha=\pi/4$.

Here we present a general result for $0<\alpha<\pi/2$ by a completely different proof with minimum prerequisite knowledge. We shall show

THEOREM 1. *Let $0<\alpha<\pi/2$. If $F\in B_{\Delta(\alpha)}$, then*

$$\iint_{\Delta(\alpha)}|F(x+iy)|^2 dxdy=\sin(2\alpha)\sum_{j=0}^{\infty}\frac{(2\sin\alpha)^{2j}}{(2j+1)!}\int_0^{\infty}x^{2j+1}|\partial^j f(x)|^2 dx, \tag{1}$$

where f stands for the trace of F on the positive real axis. Conversely, if f is a smooth function on the positive real axis for which the right hand side of (1) *is finite, then f has an analytic continuation $F\in B_{\Delta(\alpha)}$ and* (1) *holds.*

It is natural to consider a counterpart of Theorem 1 for the Szegö space

$$S_{\Delta(\alpha)}=\left\{F; F \text{ is analytic on } \Delta(\alpha),\ \sup_{|\theta|<\alpha}\int_0^{\infty}|F(re^{i\theta})|^2 dr<\infty\right\},$$

which is normed by the square root of $\int_{\partial\Delta(\alpha)}|F(z)|^2|dz|$ with $F(z)$ being the nontangential boundary values of F on $\partial\Delta(\alpha)$. We shall prove