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Geometric 4 manifolds in the sense of Thurston and Seifert 4 manifolds II

Dedicated to Professor Akio Hattori on his 60th birthday

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This paper is a sequel to [22] and characterizes the geometric 4 manifolds of type $S^3\times E$, $S^2\times E^2$, $H^2\times E^2$, and $\widetilde{SL}_2\times E$ in terms of the Seifert 4 manifolds over the 2 orbifolds which are either spherical, bad, or hyperbolic. In [22] we discussed the relations between the Seifert 4 manifolds over the euclidean 2 orbifolds and the geometries of type E^4, $Nil^3\times E$, Nil^4, and $Sol^3\times E$. Here we call a 4 manifold S a Seifert 4 manifold if S has a structure of a nonsingular fibered orbifold over a 2 orbifold B with general fiber a 2 torus T^2 as in [22]. The topology of S can be described by the Seifert invariants defined in [22]. We will recall their descriptions briefly in § 1 and § 5 when B is either spherical, bad or hyperbolic. If all the monodromies are trivial (including one more case when the base is spherical) we can define the rational euler class e which is a rational number or a pair of rational numbers (§ 1 and § 5). Then the main results (Theorems A and B) which provide the complementary part of Theorems A and B in [22] can be stated as follows.

THEOREM A. *Let S be a closed orientable 4-manifold.* (1) *S is a Seifert 4 manifold over a spherical or bad 2 orbifold whose rational euler class is zero (resp. nonzero) if and only if S is geometric of type $S^2\times E^2$ (resp. $S^3\times E$).* (2) *S is geometric of type $S^3\times E$ if and only if S is diffeomorphic to a bundle over S^1 with fiber a spherical 3 manifold. S is geometric of type $S^2\times E^2$ if and only if S is diffeomorphic to a nonsingular fibered orbifold with general fiber S^2 over a euclidean orbifold B' where B' is either the torus T^2, the Klein bottle K, the annulus A or the Möbius band M.*

See § 3, § 4 for the details of the correspondences in Theorem A. We will determine exactly when the Seifert 4 manifolds of the above classes admit complex structures in § 3 Corollary 9, § 4 Corollary 13. In Corollary 9 we give the explicit correspondence between the Seifert fibrations and the bundle structures over S^1 of the Hopf surfaces since some of them were missing in [9] (see [10]) and since not every bundle over S^1 with fiber a spherical 3 manifold