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Homogeneity and complete decomposability of torsion free knot modules

Dedicated to Professor Fujitsugu Hosokawa on his 60th birthday

By Katsuyuki YOSHIKAWA

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Let Λ be the integral group ring of the infinite cyclic group $\langle t: \rangle$. A Λ-module A is called a *knot module* if A is finitely generated over Λ and $t-1$ induces an automorphism of A. The purpose of this paper is to generalize results of E. S. Rapaport [8], R. H. Crowell [**2**] and D. W. Sumners [**9**] on knot modules. M. Kervaire [**5**] showed that the Z-torsion part T of a knot module A is a finite Λ-submodule. It follows from [**3**, vol. 2, p. 187] that A splits as an abelian group, i.e., $A \cong_Z F \oplus T$, where $F=A/T$. The Z-torsion part of a knot module has been completely determined. That is, a finite abelian group T is isomorphic to the Z-torsion part of some knot module if and only if the number of factors isomorphic to Z_{2^i} in the 2-primary component of T is not equal to one for any positive integer i (cf. [**4**], [**6**]). On the other hand, it still remains open to characterize the Z-structure of a Z-torsion free knot module. In this paper, we investigate two classes of Z-torsion free knot modules; one is homogeneous and the other is completely decomposable. Using our result, we can find an answer to Sumners's question [**9**, p. 84] for models of Z-torsion free knot modules.

Throughout this paper (unless otherwise specified), all groups will be Z-torsion free abelian and all Λ-modules will be Z-torsion free knot modules.

1. Introduction.

A polynomial $f(t)$ of Λ is *primitive* if all its coefficients are relatively prime. Let A be a Λ-module. Then $A \otimes_Z Q$ is a finitely generated Γ-module, where $\Gamma = \Lambda \otimes_Z Q$. Therefore, since Γ is a principal ideal domain, we have

$$A \otimes_Z Q \cong_\Gamma \Gamma/(\lambda_1) \oplus \cdots \oplus \Gamma/(\lambda_k).$$

In the above decomposition, one can take the λ_i to be primitive elements of Λ such that $\lambda_{i+1} | \lambda_i$ in Λ, $i=1, \cdots, k-1$. We call $\{\lambda_i\}_{i=1}^k$ the (*rational*) *polynomial*