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A characterization of the association schemes of Hermitian forms

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Let $Y=(X, \{R_i\}_{0\leq i\leq d})$ be an association scheme whose parameters coincide with those of the association scheme $\mathrm{Her}(d, q)$ of Hermitian forms in d-dimensional space over the field $GF(q^2)$. Suppose that every edge of the distance-regular graph $\Gamma=(X, R_1)$ is contained in a clique of size q. It is shown that if $d\geqq 3$ then Y is isomorphic to $\mathrm{Her}(d, q)$. In the case $d=2$ a generalized quadrangle with the parameters (q, q^2) is reconstructed from Y.

1. Introduction.

The present paper is a continuation of [**IS1**] where the particular case $q=2$ was treated completely and some results concerning the general situation were proved. A detailed discussion of the schemes of Hermitian forms is contained in [**BI**], [**BCN**] and [**IS1**]. Here we give only the necessary definitions.

Let X be the set of all Hermitian forms (singular or nonsingular) in the space of dimension d over $GF(q^2)$ and $R_0, R_1, \cdots, R_d$ be the relations on X defined as follows

$$(x, y)\in R_i \quad \text{if and only if } \mathrm{rank}(x-y)=i,\ 0\leqq i\leqq d\,.$$

Then $Y=(X, \{R_i\}_{0\leq i\leq d})$ is a (P and Q)-polynomial association scheme known as the scheme of Hermitian forms $\mathrm{Her}(d, q)$. The distance-regular graph $\Gamma=(X, R_1)$ related to the scheme $\mathrm{Her}(d, q)$ has the following parameters:

$$\begin{aligned} b_i&=(q^{2d}-q^{2i})/(q+1)\,,\\ c_i&=(q^{i-1}(q^i-(-1)^i))/(q+1)\,,\\ a_i&=(q^{2i}-q^{i-1}(q^i-(-1)^i)-1)/(q+1)\,. \end{aligned} \tag{1}$$

Apparently for the first time these facts were proved in [**Wan**].

The main result of the paper is the following.

THEOREM A. *Let Γ be a distance-regular graph of diameter $d\geqq 2$, whose*