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Signature defects and eta invariants of Picard modular cusp singularities

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Introduction.

A compact oriented framed manifold (N, α) of dimension $4k-1$ has an invariant called *signature defect* defined by Hirzebruch in [**Hi**] as follows: Since N is framed, there exists a compact oriented $4k$ dimensional manifold M with $\partial M=N$ and the tangent bundle of M restricted to ∂M is trivialized. Thus we can define the Pontrjagin classes of M as relative classes p_j in $H^{4j}(M, \partial M; \boldsymbol{Z})$. Hirzebruch defined the signature defect as

$$\sigma(N, \alpha):=L_k(p_1, \cdots, p_k)[M, \partial M]-\mathrm{sign}(M, \partial M),$$

where $L_k(p_1, \cdots, p_k)\in H^{4k}(M, \partial M; \boldsymbol{Q})$ is the Hirzebruch L-polynomial with respect to p_j's, $[M, \partial M]$ is the fundamental class of $(M, \partial M)$ and $\mathrm{sign}(M, \partial M)$ is the signature of the bilinear form on $H^{2k}(M, \partial M; \boldsymbol{R})$ defined by cup product.

In [**Hi**] Hirzebruch showed that a Hilbert modular cusp singularity (X, p) has a compact neighborhood V of p such that the boundary ∂V is framed and conjectured that the signature defect of the singularity is equal to the special value of Shimizu's L-function. He proved the conjecture in the 2-dimensional case.

On the other hand, Atiyah, Patodi and Singer [**APS1**] defined the eta invariants of first order self-adjoint elliptic differential operators on compact manifolds, and derived the index theorem for manifolds with boundary. Their index theorem says that the difference between the integral of the closed differential form representing the L-genus and $\mathrm{sign}(M, \partial M)$ is equal to the eta invariant of the tangential signature operator on the boundary manifold ∂M.

By using the index theorem for manifolds with baundary in [**APS1**] Atiyah, Donnelly and Singer proved Hirzebruch's conjecture in general ([**ADS1**], [**ADS2**]). And Müller also proved it ([**Mu2**]).

The purpose of this paper is to study the signature defects of Picard

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