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Geometric 4-manifolds in the sense of Thurston and Seifert 4-manifolds I

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The notion of geometric manifolds introduced by Thurston plays a fundamental role in the 3-dimensional topology ([**14**], [**15**]). There are just eight 3-dimensional geometries and the six among these correspond to the Seifert 3-manifolds ([**13**]). On the other hand the geometries in dimension 4 were classified by Filipkiewicz [**6**] and the geometric structures of 4-manifolds, in particular those of complex surfaces were studied by Wall ([**18**], [**19**]). For example there is a correspondence between the elliptic surfaces without singular fibers and the certain geometries analogous to that for Seifert 3-manifolds ([**18**], Theorem 7-4). The purpose of this paper (together with Part II [**17**]) is to give the correspondence between Seifert 4-manifolds which are not necessarily complex surfaces and the following eight geometries; E^4, $Nil^3\times E$, Nil^4, $Sol^3\times E$, $S^3\times E$, $S^2\times E^2$, $H^2\times E^2$ and $\widetilde{SL}_2\times E$. Here a Seifert 4-manifold is a 4-manifold which has a structure of a fibered orbifold over a 2-orbifold with general fiber a 2-torus (see § 1). We will characterize the closed orientable geometric 4-manifolds of the above eight types in terms of the Seifert 4-manifolds (Theorems A and B) and will also give the topological classification of such 4-manifolds (cf. Part II [**17**]). The notion of the Seifert 4-manifolds of the above types coincides with that in the usual sense studied in [**21**], [**22**] for example if the base orbifolds have no reflectors. We need to take account of the cases when the bases have (corner) reflectors to prove the converse direction (Theorem B) of the above correspondence. We only consider the closed orientable ones and then a fiber over a (corner) reflector point is a Klein bottle multiply covered by the general fiber. The topology of Seifert 4-manifolds of this type can be described by a series of invariants (which we call the Seifert invariants) analogous to those for Seifert 3-orbifolds ([**3**], [**5**], [**14**]) and will be explained in § 1. In the present paper we will restrict our attention to the cases with euclidean base orbifolds and the corresponding four types of geometries. The other cases will be treated in Part II ([**17**]). But for convenience we will give the results in full generality in the following section.