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Canonical stratification of non-degenerate complete intersection varieties

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§ 1. Introduction.

This paper is a continuation of [19] and [21]. Unless otherwise stated, we use the same notation as in [21] and we assume the results in [19] and [21]. (This paper is a revised version of [20]. § 7 and Appendix B in § 10 are added.) Let V be a germ of complete intersection variety at the origin of $\boldsymbol{C}^n$. The singularity of V is not necessarily isolated. The purpose of this paper is to describe the canonical toroidal resolution of V, the limits of the tangent spaces and to construct a canonical Whitney b-regular stratification on V under a certain condition (IND-condition). It is very important to get a regular stratification to study non-isolated singularities. In [4], J. Damon considered the topological stability problem of a family of complex hypersurfaces $V_t=\{f_t(\boldsymbol{z})=0\}$ with non-isolated singularities using the vector field argument. He showed that the topological types of V_t do not change if the Newton boundary is strongly non-degenerate and $\Gamma(f_t)=\Gamma(f_0)$. One motivation of this research is to understand this property from the stratification point of view. In [19], we have showed the existence of a canonical stratification for a *good* hypersurface. However, in the process of the stratification of a hypersurface with non-isolated singularities, it turns out that the stratification of a hypersurface which is not good involves the stratification of the complete intersection varieties. See Example (9.3). Thus we consider the following situation. Let $V=\{z\in\boldsymbol{C}^n;\ f_1(\boldsymbol{z})=\cdots=f_\alpha(\boldsymbol{z})=0\}$ and let $V^*=V\cap\boldsymbol{C}^{*n}$ where $f_1, \cdots, f_\alpha$ are analytic functions defined in a neighborhood of the origin. We assume that V is a complete intersection variety with the inductive non-degeneracy condition. (See § 6 for the definition.) Let I be a subset of $\{1, \cdots, n\}$ and let $V^{*I}=V\cap\boldsymbol{C}^{*I}$ where $\boldsymbol{C}^{*I}=\{\boldsymbol{z};\ z_i\neq 0\Leftrightarrow i\in I\}$. Let V_{pr} be the closure of V^* in $\boldsymbol{C}^n$ and let $V_{pr}^{*I}=V_{pr}\cap\boldsymbol{C}^{*I}$. Note that $V_{pr}^{*I}\subset V^{*I}$.

In § 3, we construct a canonical toroidal resolution of V_{pr}. In § 4, we study the geometry of V_{pr}^{*I}. We introduce the concept of the *I-primary boundary components* which play an important role for the stratification of V. Its rough description is as follows. Let $P={}^t(p_1, \cdots, p_n)$ be a positive rational dual vector and let $I=\{i;\ p_i=0\}$. Let $f_{1P}, \cdots, f_{\alpha P}$ be the face functions with respect to P