J. Math. Soc. Japan
Vol. 42, No. 1, 1990

# Harmonic mappings from $R^m$ into an Hadamard manifold

By Atsushi TACHIKAWA

(Received Dec. 26, 1988)

## 0. Introduction.

Let $M$, $N$ be Riemannian manifolds. Given a $C^1$-mapping $U: M \to N$ we define the *energy density* $e(U)(x)$ at $x \in M$ by

$$e(U)(x) = \frac{1}{2} |dU(x)|^2,$$

where $|\ \ |$ denotes the norm induced from the tensor product norm on $T_x^* M \otimes T_{U(x)} N$. For a bounded domain $\Omega$ of $M$, we define the energy of $U: M \to N$ on $\Omega$ by

$$E(U;\ \Omega) = \int_\Omega e(U) d\mu,$$

where $d\mu$ stands for the volume element on $M$. A mapping $U: M \to N$ is said to be *harmonic* if it is of class $C^2$ and satisfies the Euler-Lagrange equation of the energy functional $E$.

The notion of harmonic mappings is an extension of the one of harmonic functions. Therefore it is natural to expect that Liouville type theorems are valid also for harmonic mappings. In fact, by S. Hildebrandt-J. Jost-K.-O. Widman [3] it has been shown that a harmonic mapping $U: M \to N$ must be a constant mapping if $M$ is *simple* and the image $U(M)$ is contained in a geodesic ball $B_r \subset N$ with $r < \pi/(2\sqrt{\kappa})$, where $\kappa$ denotes the maximum of the sectional curvatures of $N$. Here, a Riemannian manifold is said to be *simple*, if it is topologically a Euclidean $m$-space $\boldsymbol{R}^m$ and furnished with a metric for which the associated Laplace-Beltrami operator is uniformly elliptic on $\boldsymbol{R}^m$. If $N$ has nonpositive sectional curvatures then we can take arbitrary $r > 0$. (See also [1] and [6].) For the case that $N$ has nonpositive sectional curvatures another type of non-existence result is shown by L. Karp [5], who proved that a non-constant harmonic mapping $U: M \to N$ defined on a complete, noncompact Riemannian manifold $M$ satisfies a certain growth order condition. This implies non-existence of harmonic mappings under some growth order condition.