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Spaces which contain a copy of the rationals

Dedicated to Professor Yukihiro Kodama on his 60-th birthday

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1. Introduction.

The main purpose of this paper is to study what kind of space contains a (closed) copy of $\boldsymbol{Q}$, where $\boldsymbol{Q}$ is the space of rationals with the usual topology. We show that every non-scattered Lašnev space contains a copy of $\boldsymbol{Q}$ and every non-scattered sequential space with character less than $\boldsymbol{b}$ contains a copy of $\boldsymbol{Q}$, where $\boldsymbol{b}$ is the minimum cardinal of an unbounded subfamily of ${}^{\omega}\omega$ (see [2]). In addition, let X_n $(n<\omega)$ be arbitrary regular topological spaces. If $\boldsymbol{Q}$ is embedded in $\prod_{n<\omega}X_n$ as a closed subset, then there exists an $n<\omega$ such that X_n contains a copy of $\boldsymbol{Q}$, where ω is the first infinite ordinal number. Moreover if we assume Martin's axiom (MA), the statement holds for any infinite cardinal number κ less than $\boldsymbol{c}$ $(=2^{\omega})$ instead of ω. The following theorems are of similar form to the last theorem.

(1) *If $\beta\omega$ is embedded in $\prod_{\alpha<\kappa}X_\alpha$ $(\kappa<\mathrm{cf}(\boldsymbol{c}))$, then there exists an $\alpha<\kappa$ such that X_α contains a copy of $\beta\omega$, where $\beta\omega$ is the Stone-Čech compactification of ω with the discrete topology.*

This theorem was proved by Malyhin [6] for the case $\kappa=\omega$ and by van Douwen-Przymusinski [3] for the other case.

(2) (Nogura-Tanaka [8]) *If S (S_2) is embedded in $\prod_{\alpha<\kappa}X_\alpha$ $(\kappa<b)$, then there exist $\alpha_1, \alpha_2, \cdots, \alpha_n$ such that $\prod_{i=1}^{n}X_{\alpha_i}$ contains a copy of S (S or S_2, respectively), where S is a sequential fan and S_2 is Arens' space* (see [1] or [8]).

We note that the closedness of embedding in our last theorem can not be dropped, because the product of infinitely many non-degenerate topological spaces contains a copy of $\boldsymbol{Q}$.

By a mapping we mean a continuous, surjective function and by a space a regular T_1 topological space.