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On the global real analytic coordinates for Teichmüller spaces

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1. Introduction.

Let G be a Fuchsian group acting on the unit disk $\boldsymbol{D}$. The group G is called type (g, m), if the quotient space $\boldsymbol{D}/G$ is conformally equivalent to a compact Riemann surface of genus g with m disjoint disks removed. Then the Euler-Poincaré characteristic $\chi(\boldsymbol{D}/G)$ is $2-2g-m$. From now on we only consider those types (g, m) satisfying $\chi(\boldsymbol{D}/G)<0$ or $2g+m\geqq 3$. A Fuchsian group is marked by choosing a system of generators. Let G be a marked Fuchsian group of type (g, m). Then all other marked Fuchsian groups of this type are considered as deformations of G and they form the Teichmüller space $\boldsymbol{T}(g, m)$. The Teichmüller space $\boldsymbol{T}(g, m)$ has the structure of a real analytic manifold of dimension $6g-6+3m$.

Keen [**5**] found that $9g-9+4m$ absolute values of traces of hyperbolic elements in a marked Fuchsian group give global real analytic coordinates for $\boldsymbol{T}(g, m)$. These absolute values have a geometric interpretation on $\boldsymbol{D}/G$ as lengths of certain closed geodesics. But this number of parameters is not minimal. Seppälä and Sorvali [8] showed that $6g-4$ multipliers (corresponding to absolute values of traces) of hyperbolic elements in a marked Fuchsian group give global real analytic coordinates for $\boldsymbol{T}(g, 0)$. Recently S. Wolpert proved the result, which is equivalent to the following: *any* $6g-6$ *absolute values of traces of elements in a marked Fuchsian group can not give global* (*even locally*) *real analytic coordinates for* $\boldsymbol{T}(g, 0)$. Hence either $6g-4$ or $6g-5$ is the minimal number of such parameters for $\boldsymbol{T}(g, 0)$.

Sorvali [**9**] showed that $6g-6+3m$ multipliers of hyperbolic elements in a marked Fuchsian group give global real analytic coordinates for $\boldsymbol{T}(g, m)$ with $gm\neq 0$. In this case this number of these parameters is minimal.

In this paper, first we show that $3m-6$ absolute values of traces of hyperbolic elements in a marked Fuchsian group give global real analytic coordinates for $\boldsymbol{T}(0, m)$ (Theorem 4.1). Next for $\boldsymbol{T}(g, 0)$, we find $6g-4$ absolute values of traces giving global real analytic coordinates by the same method used in the