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Simply connected 4-manifolds of second betti number 1 bounded by homology lens spaces

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§1. Introduction.

An oriented closed 3-manifold M is called a *homology lens space* if its first integral homology group $H_1(M; \boldsymbol{Z})$ is isomorphic to a finite cyclic group $\boldsymbol{Z}/p\boldsymbol{Z}$ for some $p \geqq 2$. By Boyer [2], for any such M, simply connected *topological* 4-manifolds of second betti number 1 bounded by M are classified by certain equivalence classes of elements of $H_1(M; \boldsymbol{Z})$ together with the Kirby-Siebenmann obstructions to smoothing (see §2). Then it arises the question which of these topological 4-manifolds cannot be given any *smooth structures*. In this paper we consider this question and give a partial answer. We mainly concern ourselves with such 4-manifolds bounded by what is called Dehn surgered 3-manifold.

Let K be a smooth knot in S^3. Then we denote by $M(K; p/q)$ the oriented closed 3-manifold obtained by p/q-Dehn surgery on K, where p and q are integers with $\gcd(p, q)=1$ and $q>0$ (see [18]). It is easy to see that the Dehn surgered 3-manifold $M(K; p/q)$ is a homology lens space with $H_1(M(K; p/q); \boldsymbol{Z})$ isomorphic to $\boldsymbol{Z}/p\boldsymbol{Z}$. If $q=1$, we can attach a 2-handle to the 4-ball D^4 along K with p-framing and we denote the resulting handlebody by $V(K; p)$. Note that $V(K; p)$ is a simply connected smooth 4-manifold of second betti number 1 whose boundary $\partial V(K; p)$ is diffeomorphic to $M(K; p/1)$. Then one of our main results of this paper is the following.

COROLLARY 3.4. *Let K be a slice knot and $p>2$ an integer, where p is even or p has some prime factor p' with $p' \equiv 3 \pmod 4$. Suppose V is a simply connected topological 4-manifold of second betti number 1 bounded by the Dehn surgered 3-manifold $M(K; p/1)$. Then V admits a smooth structure if and only if it is homeomorphic to the handlebody $V(K; p)$.*

We also show, using the topological classification due to Boyer, that there do exist many simply connected topological 4-manifolds of second betti number

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