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Resolvent estimates at low frequencies and limiting amplitude principle for acoustic propagators

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Introduction.

In the present paper we study the low frequency behavior of resolvents for perturbed acoustic operators with perturbations decreasing slowly at infinity and, as an application, we prove the principle of limiting amplitude for such operators.

We work in the 3-dimensional space R_x^3, $x=(x_1, x_2, x_3)$, and consider the following equation:

$$(0.1) \qquad (\partial/\partial t)^2 w = a(x)^2 \rho(x) \nabla \cdot (1/\rho(x)) \nabla w .$$

As is well known, this equation governs the propagation of acoustic waves in an inhomogeneous medium with a local speed of sound $a(x)>0$ and an equilibrium density $\rho(x)>0$ which vary with $x \in R_x^3$. We deal with equation (0.1) under a Hilbert space formulation. First we assume that:

$$(a.0) \qquad 1/c < a(x) < c ,$$

$$(\rho.0) \qquad 1/c < \rho(x) < c$$

for some $c>1$ and

(ρ.1) $\rho(x)$ is of C^1-class with bounded derivatives.

We now define the acoustic operator L as

$$(0.2) \qquad L = -a(x)^2 \rho(x) \nabla \cdot (1/\rho(x)) \nabla .$$

Under the above assumptions, the operator L is symmetric in the Hilbert space $L^2(R_x^3; E(x)dx)$ with $E=a(x)^{-2}\rho(x)^{-1}$ and it admits a unique self-adjoint realization. We denote by the same notation L this realization and by $R(z; L)$ the resolvent of L; $R(z; L)=(L-z)^{-1}$, $\operatorname{Im} z \neq 0$. As is easily seen, L is positive (zero is not an eigenvalue) and the domain of L is given by $D(L)=H^2(R_x^3)$, $H^s(R_x^3)$ being the Sobolev space of order s. We further assume that the inhomogeneous medium under consideration is homogeneous at infinity. (This assumption will be made clear below.) Under suitable assumptions on the behavior as $|x| \to \infty$ of $a(x)$ and $\rho(x)$, we know that L has no eigenvalues and