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Some properties for the measure-valued branching diffusion processes

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1. Introduction.

The purpose of this paper is to investigate some fundamental properties for an occupation time of a measure-valued branching diffusion process $X(t)$. The process $X(t)$ arises as a high density limit of a critical branching Brownian motion on $\boldsymbol{R}^d$ (see Dawson [1] and Watanabe [7]), hence $X(t)$ may be considered as a model describing an evolution of population with spatial migration.

One of the most important problems is concerned with the limiting distribution of the process $X(t)$ as $t\rightarrow\infty$. It is well-known that if the initial state $X(0)$ is a finite measure, then the total mass process of $X(t)$ is equivalent to a one-dimensional continuous state critical branching process and hence extinction occurs almost surely. But if $X(0)$ has an infinite total mass, then interesting phenomena arise. Namely, assuming that $X(0)$ is the Lebesgue measure on $\boldsymbol{R}^d$, Dawson [1] proved the following:

(i) If $d\leqq 2$, then $X(t)$ converges vaguely to the zero measure as $t\rightarrow\infty$ in probability.

(ii) If $d\geqq 3$, then the distribution of $X(t)$ converges weakly to a non-trivial stationary distribution as $t\rightarrow\infty$.

Furthermore, under the same initial condition, Iscoe [3] obtained the following limit theorems for the occupation time process $Y(t)=\int_0^t X(s)ds$.

(iii) If $d=1$, then $P(\lim_{t\rightarrow\infty} Y(t, K)<\infty)=1$ for every compact set K.

(iv) If $d=2$, then $P(\lim_{t\rightarrow\infty} Y(t, G)=\infty)=1$ for every non-empty open set G.

(v) If $d\geqq 3$, then $P(\lim_{t\rightarrow\infty} Y(t)/t=\lambda(\text{vaguely}))=1$, where λ denotes the Lebesgue measure on $\boldsymbol{R}^d$.

However, since the above results (iii) and (iv) seem rather crude, we would like to investigate more detailed properties for the occupation time process $Y(t)$.

It is well known that the Brownian local time is often used to characterize the limiting process concerning an occupation time of a one-dimensional Brownian

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