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Artin-Schreier coverings of algebraic surfaces

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Introduction.

Let k be an algebraically closed field of characteristic $p>0$ and let X be a nonsingular projective surface defined over k. An *Artin-Schreier covering* of X is a finite morphism $\pi: Y \to X$ from a normal surface Y onto X such that the field extension $k(Y)/k(X)$ is an Artin-Schreier extension. It is well-known that $k(Y)$ is expressed as $k(Y)=k(X)(\xi)$ with $\xi^p-\xi=f$ and $f \in k(X)$. Since $k(Y)/k(X)$ is a Galois extension with the Galois group $G \cong \boldsymbol{Z}/p\boldsymbol{Z}$, G acts on Y so that $X \cong Y/G$. In order to study Artin-Schreier coverings, we have to consider whether or not there exists an affine open covering $\mathfrak{U}=\{U_\lambda\}$ such that $\pi^{-1}(U_\lambda) = \operatorname{Spec} \mathcal{O}_X(U_\lambda)[\xi_\lambda]/(\xi_\lambda{}^p - s_\lambda \xi_\lambda - t_\lambda)$ with $s_\lambda, t_\lambda \in \mathcal{O}_X(U_\lambda)$. In general, this assertion does not hold (cf. Example 1.5). Under the above circumstance, we shall define an Artin-Schreier covering of simple type (see §1 for the definition), for which the assertion holds. From the definition, every Artin-Schreier covering in characteristic 2 is of simple type.

This article consists of three parts. In Section 1, we consider Artin-Schreier coverings of simple type and give some formulas to compute invariants in the case of nonsingular coverings. In Section 2, we assume that the characteristic is 2 and consider a resolution of singularities for Artin-Schreier coverings with nonsingular branch locus. We give some formulas to compute invariants of nonsingular models of coverings. In Section 3, we consider smooth Artin-Schreier coverings of simple type with ample branch loci which satisfy extra conditions. Especially, we shall determine such coverings with $\kappa=-\infty$, 0, and 1.

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§1. Artin-Schreier coverings of simple type.

Let X be a nonsingular projective surface and let $\pi: Y \to X$ be an Artin-Schreier covering. Since Y is a Cohen-Macaulay scheme and X is regular, π is a flat morphism. Hence $\pi_* \mathcal{O}_Y$ is a locally free $\mathcal{O}_X$-algebra. Moreover,