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Collapsing Riemannian manifolds to ones with lower dimension II

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§ 0. Introduction.

The purpose of this paper is to investigate the phenomena that a sequence of Riemannian manifolds M_i converges to ones with lower dimension, N, with respect to the Hausdorff distance, which is introduced in [**11**]. We have studied this phenomena in [**7**] and proved there that M_i is a fibre bundle over N with infranilmanifold fibre. In this paper, we study which fibre bundle it is, and give a necessary and sufficient condition. We will describe it in Theorem 0-1 and 0-7.

THEOREM 0-1. *Let M_i be a sequence of $n+m$-dimensional compact Riemannian manifolds and N be an n-dimensional compact Riemannian manifold. Assume*

(0-2-1) *M_i converges to N with respect to the Hausdorff distance,*

(0-2-2) $|$*sectional curvature of* $M_i| \leqq 1$.

Then, for sufficiently large i, there exists a map $\pi_i: M_i \to N$ such that the following hold.

(0-3-1) *π_i is a fibre bundle.*

(0-3-2) *$\pi_i^{-1}(p)=G/\Gamma$, where G is a nilpotent Lie group and Γ is a discrete group of affine transformations of G satisfying $[\Gamma: G\cap\Gamma]<\infty$. Here we put the (unique) connection on G which makes all right invariant vector field parallel, and G is regarded to be a group of affine transformations on G by right multiplication.*

(0-3-3) *The structure group of π_i is contained in the skew product of $C(G)/(C(G)\cap\Gamma)$ and* $\mathrm{Aut}\,\Gamma$, *where $C(G)$ denotes the center of G.*

REMARK 0-4. Statements (0-3-1) and (0-3-2) were proved in [**7**].

REMARK 0-5. [**7**, 0-1-3] also holds. Namely π_i is an almost Riemannian submersion in the sense stated there.

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