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A note on Martin boundary of angular regions for Schrödinger equations

By Toshimasa TADA

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We denote by Ω the punctured unit disk $0<|z|<1$ and consider the Martin compactification Ω_P^* ([**4**, p. 166]) of Ω with respect to a Schrödinger equation

$$(1) \qquad (-\Delta+P(z))u(z)=0 \qquad \left(\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2},\ z=x+yi\right)$$

with its potential P on Ω. The potential P on Ω is assumed to be nonnegative and locally Hölder continuous on $0<|z|\leqq 1$. We also consider the Martin compactification A_P^* of an angular region A with radius 1 and vertex at the origin $z=0$ with respect to (1). Let $\bar{\Omega}$ and $\bar{A}$ be the Euclidean closures of Ω and A, respectively. One might ask the following

QUESTION 1. *Does $A_P^*=\bar{A}$ for all angular regions A imply $\Omega_P^*=\bar{\Omega}$?*

Here the equality $\Omega_P^*=\bar{\Omega}$ ($A_P^*=\bar{A}$, resp.) means that the identity mapping of Ω (A, resp.) can be extended to a homeomorphism of Ω_P^* (A_P^*, resp.) onto $\bar{\Omega}$ ($\bar{A}$, resp.).

For a point p in the Euclidean boundary $\partial\Omega$ (∂A, resp.) of Ω (A, resp.), we denote by $\Omega_P^*(p)$ ($A_P^*(p)$, resp.) the set of all Martin boundary point ζ^* of Ω (A, resp.) for which there exists a sequence $\{\zeta_n\}_1^\infty$ in Ω (A, resp.) converging to p with respect to the Euclidean topology and at the same time converging to ζ^* with respect to the Martin topology. We call $\Omega_P^*(p)$ ($A_P^*(p)$, resp.) the Martin boundary of Ω (A, resp.) over p. We also denote by $\Omega_{P,1}^*(p)$ ($A_{P,1}^*(p)$, resp.) the set of Martin minimal boundary points over p, i.e. the subset of $\Omega_P^*(p)$ ($A_P^*(p)$, resp.) consisting of minimal points. In terms of $\Omega_{P,1}^*(0)$ and $A_{P,1}^*(0)$, Question 1 can be reformulated as

QUESTION 2. *Does $A_{P,1}^*(0)=\{$one point$\}$ for all angular regions A imply $\Omega_{P,1}^*(0)=\{$one point$\}$?*

Since P is locally Hölder continuous apart from the origin, we have $\Omega_P^*-\Omega_P^*(0)=\bar{\Omega}-\{0\}$ and $A_P^*-A_P^*(0)=\bar{A}-\{0\}$ (cf. [**1**]). By an argument similar to that

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