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Remarks on the L^2-cohomology of singular algebraic surfaces

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§ 1. Introduction.

Let X be a normal singular algebraic surface (over $\boldsymbol{C}$) embedded in the projective space $\boldsymbol{P}^N(\boldsymbol{C})$ and let S be its singularity set, which consists of isolated singular points. By restricting the Fubini-Study metric of $\boldsymbol{P}^N(\boldsymbol{C})$ to $\mathcal{X}=X-S$, we obtain an incomplete Riemannian manifold $(\mathcal{X}, g)$. Then Hsiang-Pati asserted in [**9**] that the L^2-cohomology $H^i_{(2)}(\mathcal{X})$ is naturally isomorphic to the dual of the middle intersection homology $IH^{\bar{m}}_i(X)$, which is a special case of the conjecture due to Cheeger, Goresky and MacPherson [**5**, § 4, Conjecture C] that it holds for any algebraic variety. However their proof has a certain gap. In this paper we will fill it. Our main result is therefore the reassertion.

THEOREM 1. *For the X, we have*

$$H^i_{(2)}(\mathcal{X}) \cong (IH^{\bar{m}}_i(X))^* . \tag{1.1}$$

As for the "non-normal" case, it can obviously be proved in the same way as Theorem 1 (in the "normal" case) by making its normalization, as asserted in [**9**, Theorem A′] —— see also Remark 3.3 in this paper.

In order to prove (1.1), we will make a good resolution $\pi: \tilde{X} \to X$ according to [**9**] and investigate the metric π^*g near the $\pi^{-1}(S)=\bigcup D_j$, (irreducible components), which is the first step. It is here that the gap seems to occur: though they regard the metric near the intersection points of the D_j as of the same type as the metric near the non-intersection points, the former one is dominated by the $W(+)$, not by the $W(-)$ which dominates the latter one (and is called "of Cheeger type" in [**9**]): see Types $(\pm)$ in § 2. And, because of the complexity of $W(+)$, we need some argument much subtler than that in [**9**].

Besides Theorem 1, there still remains the following problem, which has a close relation with Theorem 1. Let d_i be the exterior derivative d acting on the smooth i-forms on $\mathcal{X}$ which and whose images by the d are both square-integrable. Also let $d_{c,i}$ be its restriction to the compactly supported smooth i-forms. Then their closures $\bar{d}_i$ and $\bar{d}_{c,i}$ must be equal to each other, that is,