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On classification of parabolic reflection groups in $SU(n, 1)$

By Shoichi KITAGAWA

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§ 0. Introduction.

To classify reflection groups is one of the most important matter in the group theory. It attracted many mathematicians. For example, finite reflection subgroups of orthogonal group $O(n)$ are classified by Coxeter [**3**] —— they are called the Coxeter groups ——, those of the unitary group $U(n)$ are classified by Shephard and Todd [**9**] —— they are called the unitary reflection groups ——, discrete cocompact reflection subgroups of the complex motion group are classified by Popov [**8**] —— they are called the crystallographic reflection groups ——, and discrete reflection subgroups of the parabolic subgroup of the special unitary group $SU(n, 1)$ of signature $(n, 1)$ are partially classified by Yoshida-Hattori [**14**] and Yoshida [**12**] —— they are called the parabolic reflection groups in $SU(n, 1)$.

This paper is devoted to the complete classification of the parabolic reflection groups in $SU(n, 1)$. The group $SU(n, 1)$ gives rise to the group $\mathrm{Aut}(D)$ of analytic automorphisms of a domain $D=\{{}^t(z, u_1, \cdots, u_m)\in \boldsymbol{C}^{m+1}; 2\,\mathrm{Im}\, z-\sum|u_j|^2>0\}$, which is projectively equivalent to the complex n-ball $B^n=\{{}^t(z_1, \cdots, z_n)\in \boldsymbol{C}^n; \sum|z_j|^2<1\}$. The parabolic subgroup G of $SU(n, 1)$ is identified with a subgroup of $\mathrm{Aut}(D)$ which leaves the point P at infinity fixed. Precisely speaking, reflection groups in question are discrete subgroups of G of locally finite covolume at P.

In § 1, we review the structure of discrete subgroup of G. The main theorem is stated in § 2. Proof is given in § 3.

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§ 1. Parabolic subgroup G.

1.1. A matrix representation of G. Let V be an $(m+1)$-dimensional complex vector space with coordinates $(z, u_1, \cdots, u_m)$. Let D be a domain in V defined as follows