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# On the boundary limits of Green potentials of functions

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## 1. Introduction.

In the half space $D=\{x=(x_1, \cdots, x_n); x_n>0\}$, $n\geqq 2$, let $G(\cdot, \cdot)$ be the Green function in $D$, that is,

$$G(x, y)=\begin{cases} |x-y|^{2-n}-|\bar{x}-y|^{2-n} & \text{if } n>2, \\ \log(|\bar{x}-y|/|x-y|) & \text{if } n=2, \end{cases}$$

where $\bar{x}=(x_1, \cdots, x_{n-1}, -x_n)$ for $x=(x_1, \cdots, x_{n-1}, x_n)$. For a nonnegative measurable function $f$ on $D$, we define

$$Gf(x)=\int_D G(x, y)f(y)dy.$$

Then it is noted (see e.g. [2; Lemma 2]) that $Gf\not\equiv\infty$ if and only if

$$\int_D (1+|y|)^{-n} y_n f(y)dy<\infty. \tag{1}$$

In this paper we study the existence of nontangential limits of $Gf$ with $f$ satisfying (1) and the additional condition:

$$\int_D y_n^{\alpha} f(y)^{n/2}\omega(f(y))dy<\infty, \tag{2}$$

where $\omega(t)$ is a positive nondecreasing function on $R^1$. In case $n\geqq 3$, $\omega$ is assumed to satisfy the following conditions:

($\omega$1) There exists a positive constant $A$ such that $\omega(2r)\leqq A\omega(r)$ for any $r>0$.

($\omega$2) $\int_1^{\infty} \omega(t)^{-1/(n/2-1)} t^{-1}dt<\infty$.

($\omega$3) $\lim_{r\to\infty} \omega(r)^{-1/(n/2-1)} \int_r^{\infty} \omega(t)^{-1/(n/2-1)} t^{-1}dt=\infty$.

As typical examples of $\omega$, we give

$$\omega(t)=[\log(2+t)]^{\delta},\ [\log(2+t)]^{n/2-1}[\log(2+(\log(2+t)))]^{\delta}, \cdots,$$

where $\delta>n/2-1$.

We say that a function $u$ on $D$ has a nontangential limit $l$ at $\xi\in\partial D$ if $u(x)$