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Asymptotic behavior of elementary solutions of transient generalized diffusion equations

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1. Introduction.

Let $\mathfrak{G}=(d/dm)(d/dx)$ be a generalized diffusion operator on an interval S and $p(t, x, y)$ the elementary solution of the generalized diffusion equation

$$\partial u(t, x)/\partial t=\mathfrak{G}u(t, x), \qquad t>0,\ x\in S, \tag{1.1}$$

in the sense of McKean [**11**]. We note that $p(t, x, y)dm(y)$ is the transition probability of the generalized diffusion process having $\mathfrak{G}$ as the generator. In this paper we study the asymptotic behavior of $p(t, x, y)$ for large t under the condition that $\mathfrak{G}$ is *transient*, i.e. $\int_0^\infty p(t, x, y)dt<\infty$, and $m(x)$ varies regularly near the end points of S.

In the previous paper [**12**], we discussed the same problem for recurrent $\mathfrak{G}$. The results there verified rigorously long time tails, i.e. $t^{-\gamma}$-decay of moments with $\gamma<1$, for multiplicative stochastic processes in statistical physics. Recently Y. Okabe [**15**] studied the asymptotic behavior of the correlation functions of stationary solutions for Stokes-Boussinesq-Langevin equations in order to observe Alder-Wainwright effect, i.e. $t^{-3/2}$-decay of velocity autocorrelation function for hard sphere. Our results here for transient $\mathfrak{G}$ give an explanation for such long time tails of the type $t^{-\gamma}$ with $\gamma\geqq 1$ from the point of view of one-dimensional generalized diffusion processes.

In [**17**] we obtained a criterion, in terms of m, for the convergence of the integral $\int_1^\infty t^\gamma p(t, x, y)dt$. By using it, we can get a rough asymptotic behavior of $p(t, x, y)$ for large time t. Namely, let $S=(l_1, l_2)$ with $-\infty\leqq l_1<l_2\leqq\infty$ and suppose that one of the following assumptions (A.1), (A.2) and (A.3) is satisfied, where $0<\rho<1$, $L(x)$ is a slowly varying function, and the symbol $a(x)\sim b(x)$ as $x\to\alpha$ stands for $\lim_{x\to\alpha}a(x)/b(x)=1$.

(A.1): $|l_i|<\infty$, $i=1, 2$, there exists the limit $\theta\equiv\lim_{x\to\infty}|m(l_2-1/x)/m(l_1+1/x)|$

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