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The modified analytic trivialization of real analytic families via blowing-ups

Dedicated to Professor Yukihiro Kodama on his 60th birthday

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Introduction.

One of the most important and interesting problems in the theory of real analytic function-germs (or singularities) is to search for "nice and natural" equivalence relations in the set of germs of analytic functions.

I am sure that the notion of blow-analytic equivalence relation defined by Professor T.-C. Kuo ([**3, 4**]), is one of them.

Let $F(x; p): (\boldsymbol{R}^n \times P, 0\times P) \longrightarrow (\boldsymbol{R}, 0)$ be an analytic function, where P is a subanalytic subset of some Euclidean space. Then, T.-C. Kuo ([**4**]) proves the classification theorem: if for fixed p, $f_p(x) := F(x; p)$ has an isolated singularity at the origin, then there exists a finite filtration $\{P^i\}$ by subanalytic subsets P^i of the parameter space P of an analytic family $F(x; p)$ such that the functions $f_p(x)$ parameterized by elements p of a connected component of P^i form a blow-analytic equivalence class.

The next problem to be considered would be the following: can we construct concretely the filtration $\{P^i\}$ of P for a given analytic family $F(x; p)$ in the classification theorem or what kind of singularities form a blow-analytic equivalence class?

Several authors studied this problem, see e.g. [**1, 3, 5**].

In [**5**], it is proved that if a real analytic family $F(x; t)$ of real analytic function-germs $f_t(x) := F(x; t): (\boldsymbol{R}^n, 0) \longrightarrow (\boldsymbol{R}, 0)$ admits a simultaneous resolution ϕ, then it admits a $\pi\circ\phi$-MAT (see the definition (1.1)), where π is a finite succession of blowing-ups with non-singular centers of $\boldsymbol{R}^n$. So, the family $f_t(x)$ forms a blow-analytic equivalence class.

In [**1**] (resp. [**3**]), it is proved that if an analytic family $F(x; t)$ is non-degenerate in some sense, it admits a π-MAT along the parameter space via the blowing-up π of $\boldsymbol{R}^n$ at the origin (resp. a so-called toroidal embedding π). Here, it should be emphasized that the mapping π is concretely constructible from the Newton boundary of $F(x; t)$.

In this paper, we also study this problem. The subblowing-ups and the blowing-ups of $\boldsymbol{R}^n$ with the ideal centers defined by families are made use of