J. Math. Soc. Japan
Vol. 40, No. 1, 1988

Expansive homeomorphisms with the pseudo-orbit tracing property on compact surfaces

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(Received June 5, 1986)
(Revised Sept. 13, 1986)

§ 0. Introduction.

It is known that compact surfaces which admit Anosov diffeomorphisms are tori, and that such diffeomorphisms on tori are topologically conjugate to toral automorphisms (see J. Franks [**3**]). The purpose of this paper is to prove in topological setting the following

THEOREM. *Let M^2 be a compact surface and $f: M^2 \to M^2$ be a homeomorphism. If f is expansive and has POTP, then f is topologically conjugate to a hyperbolic toral automorphism.*

REMARK. Let M^2 be as in Theorem. It is known (see T. O'Brien and W. Reddy [**9**]) that if M^2 is orientable and has positive genus, then M^2 admits an expansive homeomorphism. Thus the assumption of POTP in our theorem can not drop.

Let (X, d) be a compact metric space and $f: X \to X$ be a homeomorphism (every homeomorphism means bijective). We say that f is *expansive* if there is $c>0$ such that when $x \neq y$, $d(f^i(x), f^i(y))>c$ for some $i \in \boldsymbol{Z}$ (c is called an *expansive constant* for f). A sequence $\{x_i\}_{i \in \boldsymbol{Z}}$ in X is a *δ-pseudo-orbit* of f if $d(f(x_i), x_{i+1})<\delta$ for all $i \in \boldsymbol{Z}$. A point x in X is said to *ε-trace* $\{x_i\}_{i \in \boldsymbol{Z}}$ if $d(f^i(x), x_i)<\varepsilon$ for all $i \in \boldsymbol{Z}$. We say that f has *POTP* if for every $\varepsilon>0$ there is $\delta>0$ such that every δ-pseudo-orbit of f is ε-traced by some point in X. We remark that expansiveness and POTP are independent of the compatible metrics used, and preserved under topological conjugacy. For materials on topological dynamics on closed manifolds, the reader may refer to A. Morimoto [8].

For $x \in X$, define the *stable* and *unstable sets* $W^s(x)$ and $W^u(x)$ as

$$W^s(x) = \{y \in X:\ d(f^n(x), f^n(y)) \to 0 \quad \text{as } n \to \infty\},$$
$$W^u(x) = \{y \in X:\ d(f^n(x), f^n(y)) \to 0 \quad \text{as } n \to -\infty\}$$

and put

$$\mathcal{F}^\sigma(X, f) = \{W^\sigma(x):\ x \in X\} \quad (\sigma = s, u).$$