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A note on the Kahn-Priddy map

Dedicated to Professor Hirosi Toda on his 60th birthday

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§ 0. Introduction.

L_p denotes the infinite dimensional lens space mod a prime p. L_p^k stands for its k-skeleton with the usual cellular decomposition $L_p=S^1\cup e^2\cup\cdots\cup e^{2n-1}\cup e^{2n}\cup\cdots$. In particular L_2^k is the real projective space P^k. Let $\lambda_k: E^{2m+1}L_p^k\to S^{2m+1}$ for $m\geqq 1$ be a mapping. Then we adopt the following definition ([7], [11], [14]): λ_k for $2p-3\leqq k\leqq 2(m+1)(p-1)-2$ is called a Kahn-Priddy map if the functional $\mathfrak{P}^1(Sq^2)$-operation of λ_k is nontrivial (resp.). From the definition, the t-fold suspension $E^t\lambda_k$ for $t\geqq 0$ is also a Kahn-Priddy map. By abuse of notation, a mapping $E^t\lambda_k$ is regarded as an element of the cohomotopy group $\pi^c(E^cL_p^k)$ for $c=t+2m+1$. λ_k' stands for the restriction $\lambda_k|E^{2m+1}L_p^{k-1}$. A stable map $E^\infty\lambda_k$ is often written $\lambda_k: L_p^k\to S^0$.

The main purpose of the present note is to determine the orders $\#(E^t\lambda_{2n})$ and $\#(E^t\lambda_{2n}')$ completely. The problem determining the order of the Kahn-Priddy map was first posed by Nishida who obtained $\#(E^\infty\lambda_{2n})=\#(E^\infty\lambda_{2n}')=p^{[n/(p-1)]}$ for an odd prime p [15]. Here $[x]$ denotes the integral part of x. In the case $p=2$, the author [12] obtained $\#(E^t\lambda_{2n})=2^{\phi(2n)}$. Here $\phi(n)$ is the number of integers in the interval $[1, n]$ congruent to 0, 1, 2 or 4 mod 8.

Nishida's method is to use the algebraic K-group of L_p^k. Our method is to follow that of [12] of which the classical KO-group of P^k [1] is used. In the case of an odd prime p, it suffices to use the K-group of L_p^k [8]. To determine the infimum of the order of a Kahn-Priddy map, we shall use the d- or e-invariant [2]. To determine the supremum, we shall use the suspension order of the stunted space L_p^{2n}/L_p^{2p-4} [4].

Let $\rho: L_p^{2n-1}\to L_p^{2n-1}/L_p^{2n-2}=S^{2n-1}$ be the canonical map. Let $\alpha_s\in\pi_{2s(p-1)-1}(S^0)$ for an odd prime p be Adams-Toda's element such that $\#\alpha_s=p$ and $e_C(\alpha_s)\equiv -1/p \bmod 1$ [2]. Then we have the following

THEOREM 1. *Let p be an odd prime, $m\geqq 1$ and $p-1\leqq n\leqq(m+1)(p-1)-1$.*

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