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Brauer-Thrall type theorem for maximal Cohen-Macaulay modules

By Yuji YOSHINO

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Let k be a perfect valuation field and let R be a local analytic k-algebra, which is always assumed to be Cohen-Macaulay. In the present paper we are interested in the category $C(R)$ of maximal Cohen-Macaulay modules. Hence the objects in $C(R)$ are finitely generated modules M with equality $\mathrm{depth}(M)=\dim(R)$. We say that $C(R)$ is of finite representation type provided that there are only finite number of isomorphic classes of indecomposable objects in $C(R)$. Analytic algebras with $C(R)$ of finite representation type are recently studied by various authors and they actually become well-understandable objects in ring theory. In fact, if k is algebraically closed of characteristic 0, then a Gorenstein algebra has $C(R)$ of finite representation type only when it is a simple hypersurface singularity [**10**]. Moreover if R has dimension 2, then the finiteness of representation type of $C(R)$ is equivalent to that R is a quotient singularity. See Artin-Verdier [**1**], Auslander [**4**] and Herzog [**14**]. In the case of dimension 1, such finiteness is characterized by the condition that R dominates a simple plane curve as is shown by Greuel-Knörrer [**13**]. See also Knörrer [**19**] and Kiyek-Steinke [**20**].

In this paper we will give a certain sufficient condition for $C(R)$ to be of finite representation type in the case R has only an isolated singularity. Precisely, if there is an upper bound for multiplicities of indecomposable modules in $C(R)$, then $C(R)$ is of finite representation type. See (1.4). This is, of course, an analogous result to Brauer-Thrall conjecture or Roiter-Auslander theorem for Artin rings. We will also show that the corresponding result of the Auslander-Reiten theory for Artin algebras is valid for the category $C(R)$. See Theorem (1.1). It should be noted that these results will fail unless R is an isolated singularity. (Cf. (1.6).)

Precise statement of our main theorem will be given in Section 1 and the subsequent sections will be devoted to a proof and an application of the theorem.

In Section 2 we will discuss a method which reduces some problems into

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