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Note on H^p on Riemann surfaces

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The purpose of this note is to prove a theorem which implies the following: Given an arbitrary open Riemann surface R and an arbitrary positive real number p. There exist a holomorphic function f on R and two subregions S and T of R with $S\cup T=R$ such that $f|S$ ($f|T$, resp.) belongs to $H^p(S)$ ($H^p(T)$, resp.) and yet f does not belong to $H^p(R)$.

1. We denote by $H^p(R)$ for a positive real number p the class of holomorphic functions f on an open Riemann surface R such that $|f|^p$ has a harmonic majorant on R. In this note we prove the following

THEOREM. *For an arbitrary holomorphic function f on an arbitrary open Riemann surface R and any positive real number p, there exist two subregions S_f and T_f of R with $S_f\cup T_f=R$ such that $f|S_f$ ($f|T_f$, resp.) belongs to $H^p(S_f)$ ($H^p(T_f)$, resp.).*

This result was originally obtained by Bañuelos and Wolff [1] when R is the unit disk. The proof will be given in nos. 2-7.

Proof of the Theorem.

2. First we fix our basic notation. We take an exhaustion $\{R_n\}_1^\infty$ of R (cf. e.g. [2]) and denote by $\{U_{nj}\}_{j=1}^{\nu_n}$ ($n=1, 2, \cdots$) the connected components of $U_n=R_{2n-1}-\bar{R}_{2n-2}$, where we set $R_0=\emptyset$. We connect U_{nj} ($j=1, \cdots, \nu_n$; $n=2, 3, \cdots$) with R_{2n-3} by a strip $V_{nj}=\phi_{nj}(D_{nj})$ in $R_{2n-2}-\bar{R}_{2n-3}$, i.e. an image of a rectangle

$$D_{nj}=\{x+yi : 0<x<1, 0<y<y_{nj}\}$$

by a conformal mapping ϕ_{nj} of a neighborhood of $\bar{D}_{nj}$ to R. We may assume that

$$\phi_{nj}([0, y_{nj}i])=\partial V_{nj}\cap\partial R_{2n-3},$$

$$\phi_{nj}([1, 1+y_{nj}i])=\partial V_{nj}\cap\partial U_{nj},$$

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