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Actions of finite groups on finite von Neumann algebras and the relative entropy

Dedicated to Professor Osamu Takenouchi on his 60th birthday

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Introduction.

Let M be a finite von Neumann algebra with a faithful normal normalized trace τ and N be a von Neumann subalgebra of M. Then, the relative entropy $H(M|N)$ is naturally defined as an extended notion of the conditional entropy in commutative cases. This relative entropy is used in Connes-Stormer's work [4] as a technical tool for finite dimensional algebras M. Recently, O. Pimsner and S. Popa have deeply studied it ([12]). One of their main results is to make clear the relationship between $H(M|N)$ and Jones' index $[M:N]$ for a type II_1 factor M and its subfactor N and give the formula on $H(M|N)$ for this pair. Another one is to compute completely the value of $H(M|N)$ for an arbitrary subalgebra N of a finite dimensional algebra M.

The aim of this paper is to give the complete formula on $H(M|M^G)$ for an arbitrary action α of a finite group G on a finite von Neumann algebra M by the following method, where M^G is the fixed point subalgebra of M under the action α.

[A] *A general case may be reduced to the case that the action α is centrally ergodic, see Proposition* 2.1.

[B] *The case where α is centrally ergodic may be reduced to the case that M is a factor, see Proposition* 2.2.

[C] *When M is a factor, $H(M|M^\alpha)$ may be computed in association with the conjugacy invariants of actions introduced and deeply studied by V. Jones* [6], *see Theorem* 2.6.

Applying these formulas, we can show the fact that $H(M|M^\alpha)\leqq\log|G|$ holds in general and we can characterize such actions α that $H(M|M^\alpha)$ attains $\log|G|$,

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