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Spectral synthesis on the algebra of Hankel transforms

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1. Introduction.

Let $\nu \geqq -1/2$. The Hankel transform of order ν is given by

$$\hat{g}(y)=\int_0^\infty g(x)J_\nu(xy)x^{\nu+1}y^{-\nu}dx, \quad y\geqq 0$$

for a function $g(x)$ on $[0, \infty)$, where $J_\nu(t)$ is the Bessel function of the first kind of order ν. Let

$$A^{(\nu)}=\left\{\hat{g}\ ;\ \int_0^\infty |g(x)|x^{2\nu+1}dx<\infty\right\},$$

and introduce a norm to $A^{(\nu)}$ by

$$\|\hat{g}\|=\frac{1}{2^\nu\Gamma(\nu+1)}\int_0^\infty |g(x)|x^{2\nu+1}dx.$$

Then the followings are known (cf. [**10**], [**7**]):

(i) $A^{(\nu)}$ consists of continuous functions on $[0, \infty)$ vanishing at infinity.

(ii) $A^{(\nu)}$ is a semisimple regular Banach algebra with the product of pointwise multiplication, and the maximal ideal space of $A^{(\nu)}$ is identified with the interval $[0, \infty)$.

Let $A(\boldsymbol{R}^n)$ be the Fourier algebra given by $A(\boldsymbol{R}^n)=\{\tilde{g}\,;\,g\in L^1(\boldsymbol{R}^n)\}$, $\|\tilde{g}\|=\|g\|_{L^1(\boldsymbol{R}^n)}$, where $\tilde{g}$ is the Fourier transform of g, that is, $\tilde{g}(\xi)=(2\pi)^{-n/2}\int_{\boldsymbol{R}^n}g(v)e^{-v\xi i}dv$, $\xi\in\boldsymbol{R}^n$. Denote by $A_r(\boldsymbol{R}^n)$ the Fourier transforms of the radial functions g, $g(v)=g(|v|)$ a.e. $v\in\boldsymbol{R}^n$, in $L^1(\boldsymbol{R}^n)$. From the well-known formula $\tilde{g}(\xi)=\hat{g}(|\xi|)$ for a radial function g, it follows that $A_r(\boldsymbol{R}^n)$ is isomorphic and isometric to $A^{(\nu)}$ if $\nu=(n-2)/2$, $n=1, 2, 3, \cdots$.

L. Schwartz [**11**] showed that the unit sphere S^{n-1} in $\boldsymbol{R}^n$ is not a set of spectral synthesis for $A(\boldsymbol{R}^n)$, $n\geqq 3$. Reiter [7] proved that, if $n\geqq 3$, then the singleton $\{y_0\}$, $y_0>0$ is not a set of spectral synthesis for $A_r(\boldsymbol{R}^n)$, that is, for $A^{(\nu)}$, $\nu=(n-2)/2$. This implies L. Schwartz's result. For, if $\tilde{g}_j\to\tilde{g}$ in $A(\boldsymbol{R}^n)$, then the Fourier transforms of the means of g_j on S^{n-1} converge to the Fourier transform of the mean of g on S^{n-1} in $A_r(\boldsymbol{R}^n)$. A. Schwartz [**10**] showed that