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On the global strong solutions of coupled Klein-Gordon-Schrödinger equations

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1. Introduction.

In this paper we will consider the following system of equations in three space dimensions:

$$id\psi/dt-A_1\psi=-\psi\phi\,, \tag{1.1}$$

$$d^2\phi/dt^2+A_2\phi=|\psi|^2\,. \tag{1.2}$$

Where A_1 and A_2 denote positive selfadjoint elliptic operators of order 2 with Dirichlet-zero conditions over a bounded or unbounded domain $\Omega\subset\boldsymbol{R}^3$. If $A_1=-\Delta$ and $A_2=-\Delta+I$, where Δ denotes the spatial Laplacian, (1.1) and (1.2) are the so called Klein-Gordon-Schrödinger (K-G-S) equations with Yukawa coupling in which ψ describes complex scalar neucleon field and ϕ describes real scalar meson field.

The first study for the K-G-S equations was done by I. Fukuda and M. Tsutsumi [7]. They considered the initial boundary value problem for the K-G-S equations under the initial conditions $\psi(0, x)=\psi_0(x)\in H_0^{1,2}(\Omega)\cap H^{3,2}(\Omega)$, $\phi(0, x)=\phi_0(x)\in H_0^{1,2}(\Omega)\cap H^{2,2}(\Omega)$, $\phi_t(0, x)=\phi_1(x)\in H_0^{1,2}(\Omega)$ and the boundary conditions $\psi(t, x)=\phi(t, x)=0$ for $x\in\partial\Omega$ and $t\in\boldsymbol{R}$. Here Ω is a bounded domain in $\boldsymbol{R}^3$ and $\partial\Omega$ is a smooth boundary of Ω. By using Galerkin's method, they proved the existence of global strong solutions of the K-G-S equations under the above conditions. The initial condition on $\psi_0(x)$ is unnatural and should be changed into the natural condition such as $\psi_0(x)\in H_0^{1,2}(\Omega)\cap H^{2,2}(\Omega)$.

The second study was done by J. B. Baillon and J. M. Chadam [2]. They proved the existence of global strong solutions of the initial value problem of the K-G-S equations under the initial conditions $\psi_0(x)\in H^{2,2}(\boldsymbol{R}^3)$ and $\phi_0(x)\in H^{2,2}(\boldsymbol{R}^3)$ and $\phi_1(x)\in H^{1,2}(\boldsymbol{R}^3)$. They obtained the above result by using L^p-L^q estimates for the elementary solution of the linear Schrödinger equation. The L^p-L^q estimates are very useful methods to the initial value problem for the K-G-S equations (see, e.g., A. Bachelot [1]). But such L^p-L^q estimates are not obtained in the case of initial boundary value problem. Therefore it does not seem that their method is directly applicable to the initial boundary value problem (1.1) and (1.2).