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Minimal 2-spheres with constant curvature in $P_n(\boldsymbol{C})$

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Introduction.

Minimal surfaces with constant curvature in real space forms have been classified completely (cf. [5], [9], [2]). A next interesting problem is to classify minimal surfaces with constant curvature in complex space forms. The purpose of this peper is to classify minimal 2-spheres with constant curvature in complex projective spaces.

Now let $S^2(c)$ be a 2-dimensional sphere with constant curvature c and $P_n(\boldsymbol{C})$ an n-dimensional complex projective space with the Fubini-Study metric of constant holomorphic sectional curvature 1. There are two typical classes of minimal isometric immersions of $S^2(c)$ into $P_n(\boldsymbol{C})$.

One is a class of holomorphic isometric imbeddings of $P_1(\boldsymbol{C})$ into $P_n(\boldsymbol{C})$ given by Calabi [4];

$$\begin{aligned} \psi_n : P_1(\boldsymbol{C}) = S^2(1/n) &\longrightarrow P_n(\boldsymbol{C}) \\ (z_0, z_1) &\longrightarrow (\sqrt{n!/(l!(n-l)!)}z_0^l z_1^{n-l})_{l=0,\dots,n}\,, \end{aligned}$$

where (z_0, z_1) is the homogeneous coordinate system of $P_1(\boldsymbol{C})$. ψ_n is called the n-th Veronese imbedding of $P_1(\boldsymbol{C})$.

The other is a class of totally real minimal isometric immersions obtained by composing a Borůvka sphere $S^2(1/2k(k+1)) \to S^{2k}(1/4)$ (cf. [1]), a natural covering $S^{2k}(1/4) \to P_{2k}(\boldsymbol{R})$ and a totally real totally geodesic imbedding $P_{2k}(\boldsymbol{R}) \to P_{2k}(\boldsymbol{C})$;

$$\mu_k : S^2(1/2k(k+1)) \longrightarrow P_{2k}(\boldsymbol{C})\,.$$

In this paper we give a family of minimal isometric immersions of 2-spheres with constant curvature into $P_n(\boldsymbol{C})$ which are not always holomorphic or totally real, using the theory of unitary representations of $SU(2)$. For $n \geqq 3$, we get examples of minimal 2-spheres with constant curvature in $P_n(\boldsymbol{C})$ which are neither holomorphic nor totally real. We will get the following:

THEOREM 1. *For any nonnegative integers n and k with $0 \leqq k \leqq n$, there exists an $SU(2)$-equivariant minimal isometric immersion*