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Fourier integral operators with weighted symbols and micro-local resolvent estimates

Dedicated to the memory of Hitoshi Kumano-go

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§ 0. Introduction.

In this paper, we investigate a calculus of Fourier integral operators with phase functions and symbols belonging to certain classes of weighted functions. As an application we give another proof of the micro-local resolvent estimates established in [3] and [4].

The phase function $\varphi(x, \xi)$ we consider satisfies

$$(0.1)\qquad |\partial_x^\alpha \partial_\xi^\beta(\varphi(x, \xi)-x\xi)| \leqq C_{\alpha\beta}\langle x\rangle^{\sigma-|\alpha|}$$

for some $0\leqq\sigma\leqq 1$ and

$$(0.2)\qquad \sum_{|\alpha+\beta|\leq l} \sup_{x,\xi}|\partial_x^\alpha\partial_\xi^\beta\vec{\nabla}_x\nabla_\xi(\varphi(x, \xi)-x\xi)| \leqq \tau$$

for some integer $l\geqq 0$ and $0\leqq\tau<1$. Namely $\varphi(x, \xi)$ is in a "neighborhood" of $x\xi=\sum_{j=1}^n x_j\xi_j$ in this sense. The symbol $p(x, \xi)$ satisfies

$$(0.3)\qquad \max_{|\alpha+\beta|\leq k} \sup_{x,\xi}\{\langle x\rangle^{-l+|\alpha|}\langle\xi\rangle^{-m}|\partial_x^\alpha\partial_\xi^\beta p(x, \xi)|\} < \infty$$

for some l, $m\in \boldsymbol{R}^1$ and an integer $k\geqq 0$. The essential feature of $\varphi(x, \xi)$ and $p(x, \xi)$ is that the decay order in x increases as the order of their derivatives with respect to x increases, which makes the asymptotic expansion of symbols with respect to x and the calculus possible. Our calculus is a version of that of families of Fourier integral operators involving the parameter $0<h<1$, which has been discussed in Kitada-Kumano-go [6]. Schematically, we can write "$\langle x\rangle^{-1}=h$". However, the details have to be studied separately.

Our main result of the calculus is the following. Let $\varphi(x, \xi)$ satisfy (0.1) and (0.2) for some τ small enough and some l large enough, and let $a(x, \xi)$ satisfy (0.3) with $l=m=0$ and be close enough to 1 with respect to the semi-norms defined through (0.3) with $l=m=0$. Then the Fourier integral operator defined by