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Movability and homotopy, homology pro-groups of Whitney continua

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0. Introduction.

By a *continuum* we mean a nonempty compact connected metric space. Let X be a continuum with metric d. By the *hyperspace* of X we mean $C(X)=\{A \mid A$ is a (nonempty) subcontinuum of $X\}$ with the Hausdorff metric H_d. Let $F_1(X)=\{\{x\} \mid x\in X\}$. A *Whitney map* for $C(X)$ is a continuous function $\omega: C(X)\rightarrow[0, \omega(X)]$ such that

(0.1) if $A\subset B$ and $A\neq B$, then $\omega(A)<\omega(B)$, and
(0.2) $\omega(\{x\})=0$ for each $\{x\}\in F_1(X)$.

In [**33**] and [**34**], H. Whitney showed that for any continuum X there exists a Whitney map ω for $C(X)$. In 1942, Kelley's important paper [**13**] appeared. J. L. Kelley was the first person to introduce Whitney map into the study of $C(X)$. After Kelley's work, several papers on Whitney maps have been written and Whitney maps have become standard tool and has since been used in almost all papers about hyperspaces (e. g., see the references).

Let $\mathfrak{P}$ be a topological property. The property $\mathfrak{P}$ is called a *Whitney property* provided whenever X has the property $\mathfrak{P}$, so does $\omega^{-1}(t)$ for any Whitney map ω for $C(X)$ and $0\leqq t<\omega(X)$. It is known that many properties are Whitney properties (e. g., see [**5**], [**8**], [**9**], [**13**], [**14**], [**15**], [**16**], [**18**], [**20**], [**23**], [**24**], [**27**], [**28**], [**29**], [**30**] and [**31**], etc.). Also, it is known that many properties are not Whitney properties (e. g., see [**3**], [**4**], [**10**], [**11**], [**18**], [**24**] and [**26**], etc.).

In [**8**], we proved that the property of being pointed 1-movable is a Whitney property. Also, in [**9**] we proved that the property of being movable is a Whitney property for $\theta(2)$-curves. Naturally, the following problem is raised: Is the property of being movable a Whitney property? In section 1, we give a negative answer to the problem. In fact, there exist a movable curve X and a Whitney map ω for $C(X)$ such that for some $0<t<\omega(X)$, $\omega^{-1}(t)$ is not 2-movable. In [**27**], J. T. Rogers, Jr. proved that if X is any continuum and ω is any Whitney map for $C(X)$, then there is an injection $\gamma^*: \check{H}^1(\omega^{-1}(t))$