J. Math. Soc. Japan
Vol. 39, No. 3, 1987

On étale $SL_2(F_p)$-coverings of algebraic curves of genus 2

By Hidenori KATSURADA

(Received April 25, 1984)
(Revised Nov. 20, 1985)

§ 0. Introduction.

Let C be a connected complete non-singular curve over an algebraically closed field k of positive characteristic p. In this paper, we shall give an upper bound for the number of finite étale Galois coverings of C whose Galois groups are isomorphic to $SL_2(F_p)$ (F_p: a finite field with p elements) when the genus of C is two.

To explain the situation, let us recall some known results. Let g be a positive integer, and let Δ_g be the group generated by $2g$-letters $a_1, \cdots, a_g, b_1, \cdots, b_g$ with one defining relation $a_1b_1a_1^{-1}b_1^{-1} \cdots a_gb_ga_g^{-1}b_g^{-1}=1$, and let $\bar{\Delta}_g$ be the pro-finite completion of Δ_g. Let C be a curve of genus g defined over k. Then it was shown by Grothendieck [3], and also by Popp [14] that there is a surjective continuous homomorphism from $\bar{\Delta}_g$ onto the algebraic fundamental group $\pi_1(C)$ of C, and that its kernel is contained in an arbitrary open normal subgroup of $\bar{\Delta}_g$ of index prime to p. Now fix a finite group G. Let $n(C, G)$ be the number of finite étale Galois coverings of C whose Galois groups are isomorphic to G, and for any compact Riemann surface R of genus g, let $N(R, G)$ be the number of finite unramified Galois coverings of R whose Galois groups are isomorphic to G. Recall that $N(R, G)$ is uniquely determined by g, and that it is equal to the number $N(g, G)$ of normal subgroups N of Δ_g satisfying $\Delta_g/N \cong G$. It follows from the above result that $n(C, G) \leqq N(g, G)$ for any curve C of genus g, and that the equality holds if the order of G is prime to p. So we naturally ask whether or not the equality holds for some curve C if the order of G is divisible by p. The answer is negative for a p-group or a meta-abelian group (for the former case, see Hasse and Witt [5], Šafarevič [15], and for the latter case, see Katsurada [7], and Nakajima [11]). However, when G is a non-solvable group of order divisible by p (for example $G=SL_2(F_{p^m})$ with $p^m \geqq 4$), it seems very difficult to obtain a reasonable upper bound for $n(C, G)$ in the general case. As an attempt, in [8] we treated the special case where $G=SL_2(F_4)$ and C is a certain hyperelliptic curve in charac-