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The maximal ideal space of the bounded analytic functions on a Riemann surface

Dedicated to Professor Yukio Kusunoki on his 60th birthday

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Introduction.

We denote by $H^{\infty}(R)$ the algebra of all bounded analytic functions on a Riemann surface R, by $\mathcal{M}(R)$ the maximal ideal space of the algebra $H^{\infty}(R)$ and by τ the canonical continuous mapping from R into $\mathcal{M}(R)$. In this note we shall answer negatively the following question (cf. [3], [4]): If $H^{\infty}(R)$ separates the points of R, does it follow that

(0.1) the mapping τ is a homeomorphism of R onto an open subset of $\mathcal{M}(R)$?

We shall show, in addition, that property (0.1) has several equivalent conditions; one of them asserts existence of a family of certain meromorphic functions on R (Theorem).

Property (0.1) is satisfied if R is an arbitrary domain on the complex plane or on any closed Riemann surface whenever $H^{\infty}(R)$ contains a nonconstant function. It is also satisfied for any Riemann surface of Parreau-Widom type (Stanton [7]). As indicated in Gamelin [4], property (0.1) has some applications ([2], [5]). For instance, one can show uniqueness (and existence by Theorem) of the Ahlfors function on R when (0.1) is valid.

Before stating the results, we fix notations. Equipped with the sup-norm $\|f\|=\sup_{a\in R}|f(a)|$, $H^{\infty}(R)$ is a Banach algebra. Let $H^{\infty}(R)^*$ be the dual space of the Banach space $H^{\infty}(R)$. One may identify the maximal ideal space $\mathcal{M}(R)$ as the set of all $\phi\in H^{\infty}(R)^*$ satisfying $\phi(fg)=\phi(f)\phi(g)$ $(f, g\in H^{\infty}(R))$ and $\|\phi\|=\phi(1)=1$. For each point $a\in R$, the point evaluation $f\to f(a)$ defines an element ϕ_a in $\mathcal{M}(R)$. A canonical map $\tau: R\to\mathcal{M}(R)$ is now defined by $\tau(a)=\phi_a$. Inheriting the weak* topology from $H^{\infty}(R)^*$, the set $\mathcal{M}(R)$ is a compact Hausdorff space and the map τ is continuous. Two points a, b of R are said to be *separated* by $H^{\infty}(R)$ if there is a function f in $H^{\infty}(R)$ with $f(a)\neq f(b)$, and *weakly separated* by $H^{\infty}(R)$ if there is a pair of functions f, g in $H^{\infty}(R)$ with $(f/g)(a)\neq(f/g)(b)$. The points of R are said to be (*weakly*) *separated* by $H^{\infty}(R)$