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On topologies of triangulated infinite-dimensional manifolds

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0. Introduction.

Consider $\boldsymbol{R}^n$ as the subset $\boldsymbol{R}^n \times \{(0, 0, \cdots)\}$ of the countable infinite product $\boldsymbol{R}^\omega$ of the real line $\boldsymbol{R}$. The set $\bigcup_{n\in\boldsymbol{N}} \boldsymbol{R}^n$ admits two different natural topologies. One is the weak topology with respect to the tower $\{\boldsymbol{R}^n\}_{n\in\boldsymbol{N}}$ and the space with this topology is called the *direct limit* of lines and denoted by $\operatorname{dir}\lim \boldsymbol{R}^n$ or simply by $\boldsymbol{R}^\infty$. Another is the relative topology inherited from the product topology of $\boldsymbol{R}^\omega$ and the space with this topology is denoted by σ, that is, σ is a subspace of the linear topological space $\boldsymbol{s}$ $(=\boldsymbol{R}^\omega)$ of all real sequences. (It is well-known that the pair $(\boldsymbol{s}, \sigma)$ is homeomorphic $(\approx)$ to the pair (l_2, l_2^f), where l_2^f is the linear span of the canonical orthonormal basis of Hilbert space l_2.) A separable topological manifold modeled on these spaces is called an *$\boldsymbol{R}^\infty$-manifold* or a *σ-manifold,* respectively. These are considered as two different topologizations on the same underlying set. The former is the direct limit of a tower of finite-dimensional (f.d.) compact metrizable spaces (compacta), that is, its topology is the weak topology with respect to the tower ([8, Prop. III. 2]). The latter is metrizable and coarser than the former. Both of these manifolds are triangulated, that is, each $\boldsymbol{R}^\infty$-manifold is homeomorphic to a simplicial complex with the weak (Whitehead) topology (cf. [**18**, Introduction]) and each σ-manifold is homeomorphic to a simplicial complex with the metric topology ([**11**, Theorem 15]). Let K be a simplicial complex and $|K| = \bigcup K$ the realization of K. By $|K|_{\mathrm{w}}$ and $|K|_{\mathrm{m}}$, we denote the spaces $|K|$ with the weak topology and the metric topology, respectively. We conjecture that *$|K|_{\mathrm{w}}$ is an $\boldsymbol{R}^\infty$-manifold if and only if $|K|_{\mathrm{m}}$ is a σ-manifold.* In this paper, we prove a half of this conjecture, that is,

THEOREM. *For a simplicial complex K, $|K|_{\mathrm{m}}$ is a σ-manifold if $|K|_{\mathrm{w}}$ is an $\boldsymbol{R}^\infty$-manifold.*

A map $f: X \to Y$ is a *fine homotopy equivalence* provided for each open cover $\mathcal{U}$ of Y there exists a map $g: Y \to X$ such that fg is $\mathcal{U}$-homotopic to id_Y and