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A stochastic solution of a high order parabolic equation

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§ 0. Introduction.

Our purpose in this paper is to solve the following initial value problem by a stochastic method, using an extension of a Girsanov type formula as in [4].

$$\frac{\partial W}{\partial t}(t, x) = (A+B)W(t, x), \qquad t>0, \ x\in \boldsymbol{R}^{d}, \tag{0.1, i}$$

$$W(0, x) = f(x), \tag{0.1, ii}$$

where

$$A = (-1)^{q-1}\rho \sum_{k=1}^{d}\left(\frac{\partial}{\partial x_k}\right)^{2q},$$

q is a natural number, and ρ is a complex number such that $\mathrm{Re}\,\rho>0$, and

$$B = \sum_{|\alpha|\leqq 2q} b_{\alpha}(x)\left(\frac{\partial}{\partial x}\right)^{\alpha},$$

$f(x)$ and $b_{\alpha}(x)$ are complex valued functions in a certain class $\mathfrak{F}^{0}(\boldsymbol{R}^{d})$ (see § 1), and $|\alpha|=\sum_{k=1}^{d}\alpha_k$ and $(\partial/\partial x)^{\alpha}=\prod_{k=1}^{d}(\partial/\partial x_k)^{\alpha_k}$ for multi index $\alpha=(\alpha_1, \cdots, \alpha_d)$. For $b_{\alpha}(x)$, $|\alpha|=2q$, we assume a sufficient condition, under which (0.1) is strongly parabolic.

As in [4], we consider *A-process*, which is a "Markov process" *related to*

$$\frac{\partial u}{\partial t}(t, x) = Au(t, x), \qquad t>0, \ x\in \boldsymbol{R}^{d}, \tag{0.2}$$

i.e., the density of the "transition probability" of the process is the fundamental solution of (0.2). In general, this transition probability is not positive even for real ρ. Therefore, if a completely additive measure related to A-process should be realized on a path space, then the measure would not be of bounded variation, shown as in [1, 2, 4]. Thus, A-process is not a Markov process in the usual sense.

In [4], we defined "stochastic integrals" of A-process, and each stochastic integral corresponds to a differential operator of order up to $2q-1$. Here we