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On the inverse scattering problem for the acoustic equation

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§1. Introduction.

The inverse problem for quantum and acoustic scatterings has been investigated extensively. Little attention, however, seems to have been paid to the inverse problem for the acoustic scattering. P. Lax and R. Phillips [**4**, p. 174], showed that the scattering operator associated with the wave equation in an exterior domain Ω (with $\partial\Omega$ smooth and bounded) whose solutions satisfy the boundary condition of being zero on $\partial\Omega$ uniquely determines the obstacle Ω^c. But in the case of a metric perturbation for the wave equation in $\boldsymbol{R}^n$ it is not known whether the scattering operator uniquely determines the metric or not. The purpose of this paper is to give an answer to an inverse problem related to this problem.

Let $g(x)$ be a C^∞-Riemannian metric on $\boldsymbol{R}^n$ $(n\geqq 2)$ satisfying $g(x)=I_n$ (the unit matrix of degree n) for $|x|\geqq r_0$ where r_0 is a positive number. Consider the scattering problem for the acoustic equation

$$(1.1)\qquad (\partial_t^2-\nabla\cdot g(x)^{-1}\nabla)\ u(t, x)=0 \qquad \text{in } \boldsymbol{R}^1\times\boldsymbol{R}^n,$$

where $\nabla={}^t(\partial_{x_1}, \cdots, \partial_{x_n})$. Let $S(s, \theta, \omega)$ be the scattering kernel for this problem. For each ω, $\theta\in S^{n-1}$, it is well known that $S(\cdot, \theta, \omega)$ is a distribution on $\boldsymbol{R}^1$ (see H. Soga [**2**], [**3**]). In what follows we adopt the following convention: $\sup \operatorname{sing\,supp} S(\cdot, \theta, \omega)=-\infty$ if $\operatorname{sing\,supp} S(\cdot, \theta, \omega)=\emptyset$. We consider the following

PROBLEM. Find an inhomogeneous media $g(x)$ from the known $\sup \operatorname{sing\,supp} S(\cdot, \theta, \omega)$.

Now let us prepare notations in order to give our answer to this problem. Let $g_e^n(x)$ be a surface of revolution on $\boldsymbol{R}^n$ with center 0 treated by H. Gluck and D. Singer [**1**] in the case that $n=2$, namely

$$(1.2)\qquad g_e^n(x)=I_n-\frac{e(|x|)}{|x|^2}x{}^tx, \qquad x={}^t(x_1, \cdots, x_n)\in\boldsymbol{R}^n\setminus 0,$$