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A sharp sufficient geometric condition for the existence of global real analytic solutions on a bounded domain

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1. Let $P(D)$ denote a linear partial differential operator with constant coefficients. Let $\Omega\subset \boldsymbol{R}^n$ be open and let $\mathcal{A}(\Omega)$ denote the space of real analytic functions on Ω. In this paper we give a result which refines the following one on the existence of global real analytic solutions abstracted from the work of Kawai [7]:

THEOREM 1. *Let $P(D)$ be locally hyperbolic and let K_ξ denote the local propagation cone of $P(D)$ corresponding to the direction ξ, which we assume can be chosen depending in an upper semi-continuous way on $\xi\in S^{n-1}$. Assume that Ω is bounded and that $\partial\Omega\times S^{n-1}$ can be covered by two closed subsets $X^\pm$ such that*

(1) $\quad (x, \xi)\in X^\pm$ *implies either* $P_m(\xi)\neq 0$ *or* $(\{x\}\pm K_\xi)\cap\Omega=\emptyset$

(with the double signs in the same order). Then we have $P(D)\mathcal{A}(\Omega)=\mathcal{A}(\Omega)$.

Our refined theorem is as follows:

THEOREM 2. *Let $P(D)$ be locally hyperbolic and $\Omega\subset \boldsymbol{R}^n$ a bounded open set. Assume that*

(2) $\quad (x, \xi)\in\partial\Omega\times S^{n-1}$ *implies either* $P_m(\xi)\neq 0$ *or* $(\{x\}+K_\xi)\cap\Omega=\emptyset$
for some choice of local propagation cone K_ξ at ξ (depending on x).

Then we have $P(D)\mathcal{A}(\Omega)=\mathcal{A}(\Omega)$.

Generically there are only two choices of local propagation cones at every ξ, namely $\pm K_\xi$. In that case the condition (2) is simply written as follows:

(2)′ $\quad (x, \xi)\in\partial\Omega\times S^{n-1}$ *implies either of*
$P_m(\xi)\neq 0,\ (\{x\}+K_\xi)\cap\Omega=\emptyset,\ (\{x\}-K_\xi)\cap\Omega=\emptyset.$

Theorem 2 improves Theorem 1 in the following points: We do not need